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Algebraic Connections and Curvature in Fibrations Bundles of Associative Algebras

Igor M. BURLAKOV

Tver State University, Zheljabova 33, Tver, 170100, Russian Federation
e-mail: don.burlakoff@yandex.ru

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Abstract

In this article fibrations of associative algebras on smooth manifolds are investigated. Sections of these fibrations are spinor, co spinor and vector fields with respect to a gauge group. Invariant differentiations are constructed and curvature and torsion of invariant differentiations are calculated.

Key words: Algebraic fibration, spinor, co spinor, vector field, field of connection, invariant differentiation, curvature, torsion.

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Fibrations of linear algebras are a specification of vector fibrations on smooth manifolds where the standard fiber is a linear algebra. Such specification allows for a smooth manifold to introduce some connection which is compatible with the algebraic structure of a standard fiber.

Let us consider an arbitrary associative unitary algebra $\mathbf{A}$, $\dim \mathbf{A} = n$, with basis space $\mathbf{T}_m$, $\dim \mathbf{T}_m = m$. Let $\mathbf{M}$ be a differentiable manifold, $\dim \mathbf{M} = m$. Denote by $\mathbf{T}_m(\boldsymbol{x})$ the tangent space in a point $\boldsymbol{x} \in \mathbf{M}$ and by $\mathbf{A}(\boldsymbol{x})$ the algebra with basis space $\mathbf{T}_m(\boldsymbol{x})$. By this we for the manifold $\mathbf{M}$ obtain a vector fiber bundle $\mathbf{AM}$, the standard fiber of which is a linear space of algebra $\mathbf{A}$ (see [1]).

However, $\mathbf{AM}$ is not only a vector space, because in every fiber $\mathbf{A}(\boldsymbol{x})$ we may define not only linear operations but also a product of vectors. Therefore it is useful to introduce for fiber bundle $\mathbf{AM}$ a special denomination *algebraic fibration* (see [2]). Herewith the module $\mathbf{A}(\mathbf{M})$ of smooth sections of algebraic fibration is an infinite algebra, the restriction of which to a point $\boldsymbol{x} \in \mathbf{M}$ coincides with algebra $\mathbf{A}(\boldsymbol{x})$. This algebra will be called a *gauge algebra* of fibration $\mathbf{AM}$ (analogously to modules of gauge field in time-space manifolds, see [3]). Elements of this algebra, i.e. sections of fibration , will be called *algebraic (gauge) fields* on manifold $\mathbf{M}$.

Herewith the algebra $\mathbf{A}(\mathbf{M})$ is unitary because an algebra $\mathbf{A}$ is unitary. Therefore the module $\mathbf{F}(\mathbf{M})$ of smooth functions on a manifold $\mathbf{M}$ is a subalgebra of the algebra $\mathbf{A}(\mathbf{M})$.

Now, let us denote by $\Re(\mathbf{A}(\mathbf{M}))$ a multiplicative group of all algebraic fields and call it by a regular group of the algebra $\mathbf{A}(\mathbf{M})$.

Let $\boldsymbol{\Phi} \in \mathbf{F}(\mathbf{M})$ be an arbitrary multiplicative function. This function defines a subgroup $\mathbf{G}_{\boldsymbol{\Phi}}(\mathbf{M}) \subset \Re(\mathbf{A}(\mathbf{M}))$, elements of which $\boldsymbol{\alpha} = \boldsymbol{\alpha}(\boldsymbol{x})$ fulfils the identity $\boldsymbol{\Phi}(\boldsymbol{\alpha}) = 1$. By this way, in a fibration $\mathbf{AM}$ we obtain a geometric structure, gauge motions of which are given by linear algebraic functions. Fields $\boldsymbol{\xi} = \boldsymbol{\xi}(\boldsymbol{x}) \in \mathbf{A}(\mathbf{M})$ which for an action of a gauge group $\mathbf{G}_{\boldsymbol{\Phi}}(\mathbf{M})$ satisfy

$$\boldsymbol{\psi}_L(\boldsymbol{\xi}(\boldsymbol{x})) = \boldsymbol{\alpha}(\boldsymbol{x}) \cdot \boldsymbol{\xi}(\boldsymbol{x}), \tag{1}$$

for any $\boldsymbol{\alpha}(\boldsymbol{x}) \in \mathbf{G}_{\boldsymbol{\Phi}}(\mathbf{M})$, are called *G-spinor fields* (analogously to spinor field in time-space manifolds, see [4]).

Fields $\boldsymbol{\eta} = \boldsymbol{\eta}(\boldsymbol{x}) \in \mathbf{A}(\mathbf{M})$ which for an action of a gauge group $\mathbf{G}_{\boldsymbol{\Phi}}(\mathbf{M})$ satisfy

$$\boldsymbol{\psi}_R(\boldsymbol{\eta}(\boldsymbol{x})) = \boldsymbol{\eta}(\boldsymbol{x}) \cdot \boldsymbol{\alpha}^{-1}(\boldsymbol{x}), \tag{2}$$

are called *G-co spinor fields* (by the same physical analogy).

Finally, fields $\boldsymbol{\zeta} = \boldsymbol{\zeta}(\boldsymbol{x}) \in \mathbf{A}(\mathbf{M})$ which for an action of a group $\mathbf{G}_{\boldsymbol{\Phi}}(\mathbf{M})$ satisfy

$$\boldsymbol{\psi}(\boldsymbol{\zeta}(\boldsymbol{x})) = \boldsymbol{\alpha}(\boldsymbol{x}) \cdot \boldsymbol{\zeta}(\boldsymbol{x}) \cdot \boldsymbol{\alpha}^{-1}(\boldsymbol{x}), \tag{3}$$

are called *G-vector fields.*

Let us consider some differentiation of fields $\boldsymbol{\xi} \in \mathbf{A}(\mathbf{M})$, i.e. a linear operator $\partial\colon \mathbf{A}(\mathbf{M}) \to \mathbf{A}(\mathbf{M})$ which satisfies the Leibnitz identity:

$$\partial(\boldsymbol{\xi} \cdot \boldsymbol{\eta}) = \partial(\boldsymbol{\xi}) \cdot \boldsymbol{\eta} + \boldsymbol{\xi} \cdot \partial(\boldsymbol{\eta}), \tag{4}$$

for any fields $\boldsymbol{\xi} = \boldsymbol{\xi}(\boldsymbol{x}), \boldsymbol{\eta} = \boldsymbol{\eta}(\boldsymbol{x}) \in \mathbf{A}(\mathbf{M})$. According to the Leibnitz identity the operator of differentiation is not invariant with respect to gauge action, generally. It means that the identities $\partial(\boldsymbol{\psi}_L(\boldsymbol{\xi}) = \boldsymbol{\psi}_L(\partial(\boldsymbol{\xi}))$, $\partial(\boldsymbol{\psi}_R(\boldsymbol{\xi}) = \boldsymbol{\psi}_R(\partial(\boldsymbol{\xi}))$, and $\partial(\boldsymbol{\psi}(\boldsymbol{\xi}) = \boldsymbol{\psi}(\partial(\boldsymbol{\xi}))$ are not satisfied for it. However, for any differentiation we may construct some new operators which will be invariant with respect to the action of the group $\mathbf{G}_{\boldsymbol{\Phi}}(\mathbf{M})$ on G-spinor, G-co spinor and G-vector fields.

For this purpose, we will in every point $\boldsymbol{x} \in \mathbf{M}$ consider an arbitrary set of differentiations $\partial_V = \partial_V(\boldsymbol{x})$. Denote by $\mathbf{D}(\boldsymbol{x})$ the linear space which is generated by such set and construct a fibration $\mathbf{DAM}$, fibers of which are Cartesian products $\mathbf{D}(\boldsymbol{x}) \times \mathbf{A}(\boldsymbol{x})$. Sections of this fibration $\boldsymbol{\Gamma}\{\partial\}$, where $\partial(\boldsymbol{x}) \in \mathbf{D}(\boldsymbol{x})$, which for an action of a gauge group $\mathbf{G}_{\boldsymbol{\Phi}}(\mathbf{M})$ satisfy

$$\boldsymbol{\psi}_C(\boldsymbol{\Gamma}\{\partial\}) = \boldsymbol{\alpha} \cdot \boldsymbol{\Gamma}\{\partial\} \cdot \boldsymbol{\alpha}^{-1} - \partial(\boldsymbol{\alpha}) \cdot \boldsymbol{\alpha}^{-1}, \tag{5}$$

are called G-connection and the set of them will be denoted by $\boldsymbol{\delta}\mathbf{A}(\mathbf{M})$.

It is clear to see that if $a, b \in \mathbf{R}$, $a + b = 1$, then for any connections $\boldsymbol{\Gamma}_1\{\partial\}, \boldsymbol{\Gamma}_2\{\partial\} \in \boldsymbol{\delta}\mathbf{A}(\mathbf{M})$ a field $a\boldsymbol{\Gamma}_1\{\partial\} + b\boldsymbol{\Gamma}_2\{\partial\}$ is also a connection.

If fields of connection are given we may construct a differential operator invariant with respect to the (gauge) motion. Especially, for any $\delta(\boldsymbol{x}) \in \mathbf{D}(\boldsymbol{x})$ we have the following theorem.

Theorem 1 (invariance theorem). *Let $\boldsymbol{\xi} = \boldsymbol{\xi}(\boldsymbol{x})$, $\boldsymbol{\eta} = \boldsymbol{\eta}(\boldsymbol{x})$ and $\boldsymbol{\zeta} = \boldsymbol{\zeta}(\boldsymbol{x})$ be arbitrary G-spinor, G-co spinor, and G-vector fields. The operators defined by the following formulas*

$$\nabla_L\{\partial\}\boldsymbol{\xi} = \partial\boldsymbol{\xi} + \boldsymbol{\Gamma}\{\partial\} \cdot \boldsymbol{\xi}, \tag{6}$$

$$\nabla_R\{\partial\}\boldsymbol{\eta} = \partial\boldsymbol{\eta} - \boldsymbol{\eta} \cdot \boldsymbol{\Gamma}\{\partial\}, \tag{7}$$

$$\tilde{\nabla}\{\partial\}\boldsymbol{\zeta} = \partial\boldsymbol{\zeta} + \boldsymbol{\Gamma}_1\{\partial\} \cdot \boldsymbol{\zeta} - \boldsymbol{\zeta} \cdot \boldsymbol{\Gamma}_2\{\partial\}. \tag{8}$$

are invariant with respect to motions of the group $\mathbf{G}_{\mathbf{\Phi}}(\mathbf{M})$.

In fact, if $\boldsymbol{\psi}_L(\boldsymbol{\xi}) = \boldsymbol{\alpha} \cdot \boldsymbol{\xi}$ then we may write:

$$\begin{aligned} &\nabla_L\{\partial\}(\boldsymbol{\psi}_L(\boldsymbol{\xi})) = \partial(\boldsymbol{\psi}_L(\boldsymbol{\xi})) + \boldsymbol{\psi}_C(\boldsymbol{\Gamma}\{\partial\}) \cdot \boldsymbol{\psi}_L(\boldsymbol{\xi}) \\ &= (\partial\boldsymbol{\alpha}) \cdot \boldsymbol{\xi} + \boldsymbol{\alpha} \cdot (\partial\boldsymbol{\xi}) + (\boldsymbol{\alpha} \cdot \boldsymbol{\Gamma}\{\partial\} \cdot \boldsymbol{\alpha}^{-1}) \cdot (\boldsymbol{\alpha} \cdot \boldsymbol{\xi}) - (\partial\boldsymbol{\alpha}) \cdot \boldsymbol{\alpha}^{-1}) \cdot (\boldsymbol{\alpha} \cdot \boldsymbol{\xi}) \\ &= \boldsymbol{\alpha} \cdot (\partial\boldsymbol{\xi}) + \boldsymbol{\alpha} \cdot \boldsymbol{\Gamma}\{\partial\} \cdot \boldsymbol{\xi} = \boldsymbol{\alpha} \cdot (\nabla_L\{\partial\}\boldsymbol{\xi}) = \boldsymbol{\psi}_L(\nabla_L\{\partial\}\boldsymbol{\xi}). \end{aligned}$$

The invariance of operators $\nabla_R\{\partial\}$ and $\tilde{\nabla}\{\partial\}$ may by proved analogously.

Operators are called *operators of invariant G-spinor, G-co spinor, and G-vector differentiation*, respectively.

In this case, if connections $\Gamma_1\{\partial\}$ and $\Gamma_2\{\partial\}$ of operator $\tilde{\nabla}\{\partial\}$ are identical, the operator of invariant G-vector differentiation is called symmetric and we denote it by

$$\nabla\{\partial\}\boldsymbol{\xi} = \partial\boldsymbol{\zeta} + \boldsymbol{\Gamma}\{\partial\} \cdot \boldsymbol{\zeta} - \boldsymbol{\zeta} \cdot \boldsymbol{\Gamma}\{\partial\} = \partial\boldsymbol{\zeta} + [\boldsymbol{\Gamma}\{\partial\}, \boldsymbol{\zeta}].$$

Let us remark that an action of an arbitrary operator of G-vector invariant differentiation $\tilde{\nabla}\{\partial\}\boldsymbol{\zeta}$ may be represented as an action of symmetric G-vector operator with a sum of anti-commutator of a given G-vector field $\boldsymbol{\xi}$ and another G-vector field. For this purpose we for the operator (8) introduce a G-connection $\boldsymbol{\Gamma}\{\partial\} = (\boldsymbol{\Gamma}_1\{\partial\} + \boldsymbol{\Gamma}_2\{\partial\})/2$ and we remark, that a difference $\boldsymbol{S}\{\partial\} = (\boldsymbol{\Gamma}_1\{\partial\} - \boldsymbol{\Gamma}_2\{\partial\})/2$ is a G-vector field (it will be called *G-torsion of a couple of G-connection* $\boldsymbol{\Gamma}_1\{\partial\}$ and $\boldsymbol{\Gamma}_2\{\partial\}$). Now we may write

$$\begin{aligned} \tilde{\nabla}\{\partial\}\boldsymbol{\zeta} &= \partial\boldsymbol{\zeta} + \boldsymbol{\Gamma}_1\{\partial\} \cdot \boldsymbol{\zeta} - \boldsymbol{\zeta} \cdot \boldsymbol{\Gamma}_2\{\partial\} \\ &= \partial\boldsymbol{\zeta} + \boldsymbol{\Gamma}_1\{\partial\} \cdot \boldsymbol{\zeta} - \boldsymbol{\zeta} \cdot \boldsymbol{\Gamma}\{\partial\} + \boldsymbol{S}\{\partial\} \cdot \boldsymbol{\zeta} + \boldsymbol{\zeta} \cdot \boldsymbol{S}\{\partial\} \\ &= \partial\boldsymbol{\zeta} + [\boldsymbol{\Gamma}\{\partial\}, \boldsymbol{\zeta}] + \langle \boldsymbol{S}\{\partial\}, \boldsymbol{\zeta} \rangle = \nabla\{\partial\}\boldsymbol{\zeta} + \langle \boldsymbol{S}\{\partial\}, \boldsymbol{\zeta} \rangle. \end{aligned}$$

For operators $\nabla_L\{\partial\}$, $\nabla_R\{\partial\}$ and $\nabla\{\partial\}$ the follownig theorems holds.

Theorem 2 (on curvature). *Let differentiations $\partial_1(\boldsymbol{x}), \partial_2(\boldsymbol{x}) \in \mathbf{D}(\boldsymbol{x})$ be given. Then commutators of invariant G-differentiations $\nabla_L\{\partial\}$, $\nabla_R\{\partial\}$, $\nabla\{\partial\}$ are reduced to linear functions coefficients of which are some G-vector fields $\mathbf{K}\{\partial_1, \partial_2\}$ depending on G-connections $\boldsymbol{\Gamma}\{\partial_1\}$, $\boldsymbol{\Gamma}\{\partial_2\}$, $\boldsymbol{\Gamma}\{[\partial_2, \partial_1]\}$.*

In fact, if $\nabla_L\{\partial\}\boldsymbol{\xi} = \partial\boldsymbol{\xi} + \boldsymbol{\Gamma}\{\partial\}\cdot\boldsymbol{\xi}$ then we obtain

$$\begin{aligned}\nabla_L\{\partial_2\}\nabla_L\{\partial_1\}\boldsymbol{\xi} &= \partial_2\partial_1\boldsymbol{\xi} + \partial_2\boldsymbol{\Gamma}\{\partial_1\}\cdot\boldsymbol{\xi} + \boldsymbol{\Gamma}\{\partial_1\}\cdot\partial_2\boldsymbol{\xi}\\ &+ \boldsymbol{\Gamma}\{\partial_2\}\cdot\partial_1\boldsymbol{\xi} + \boldsymbol{\Gamma}\{\partial_2\}\cdot\boldsymbol{\Gamma}\{\partial_1\}\cdot\boldsymbol{\xi},\\ \nabla_L\{\partial_1\}\nabla_L\{\partial_2\}\boldsymbol{\xi} &= \partial_1\partial_2\boldsymbol{\xi} + \partial_1\boldsymbol{\Gamma}\{\partial_2\}\cdot\boldsymbol{\xi} + \boldsymbol{\Gamma}\{\partial_2\}\cdot\partial_1\boldsymbol{\xi}\\ &+ \boldsymbol{\Gamma}\{\partial_1\}\cdot\partial_2\boldsymbol{\xi} + \boldsymbol{\Gamma}\{\partial_1\}\cdot\boldsymbol{\Gamma}\{\partial_2\}\cdot\boldsymbol{\xi},\\ \nabla_L\{[\partial_2,\partial_1]\}\boldsymbol{\xi} &= \partial_2\partial_1\boldsymbol{\xi} - \partial_1\partial_2\boldsymbol{\xi} + \boldsymbol{\Gamma}\{[\partial_2,\partial_1]\}\cdot\boldsymbol{\xi}.\end{aligned}$$

Therefore

$$(\nabla_L\{\partial_2\}\nabla_L\{\partial_1\} - \nabla_L\{\partial_1\}\nabla_L\{\partial_2\} - \nabla_L\{[\partial_2,\partial_1]\})\boldsymbol{\xi} = \mathbf{K}\{\partial_1,\partial_2\}\cdot\boldsymbol{\xi},$$

where

$$\mathbf{K}\{\partial_1,\partial_2\} = \partial_2\boldsymbol{\Gamma}\{\partial_1\} - \partial_1\boldsymbol{\Gamma}\{\partial_2\} + \boldsymbol{\Gamma}\{\partial_2\}\cdot\boldsymbol{\Gamma}\{\partial_1\} - \boldsymbol{\Gamma}\{\partial_1\}\cdot\boldsymbol{\Gamma}\{\partial_2\} - \boldsymbol{\Gamma}\{[\partial_2,\partial_1]\}.$$

By na analogical way, we may prove

$$(\nabla_L\{\partial_2\}\nabla_L\{\partial_1\} - \nabla_L\{\partial_1\}\nabla_L\{\partial_2\} - \nabla_L\{[\partial_2,\partial_1]\})\boldsymbol{\xi} = \mathbf{K}\{\partial_1,\partial_2\}\cdot\boldsymbol{\xi},$$

where

$$\mathbf{K}\{\partial_1,\partial_2\} = \partial_2\boldsymbol{\Gamma}\{\partial_1\} - \partial_1\boldsymbol{\Gamma}\{\partial_2\} + \boldsymbol{\Gamma}\{\partial_2\}\cdot\boldsymbol{\Gamma}\{\partial_1\} - \boldsymbol{\Gamma}\{\partial_1\}\cdot\boldsymbol{\Gamma}\{\partial_2\} - \boldsymbol{\Gamma}\{[\partial_2,\partial_1]\}.$$

By na analogical way, we may prove

$$(\nabla_R\{\partial_2\}\nabla_R\{\partial_1\} - \nabla_R\{\partial_1\}\nabla_R\{\partial_2\} - \nabla_R\{[\partial_2,\partial_1]\})\boldsymbol{\eta} = -\boldsymbol{\eta}\cdot\mathbf{K}\{\partial_1,\partial_2\},$$

and

$$(\nabla\{\partial_2\}\nabla\{\partial_1\} - \nabla\{\partial_1\}\nabla\{\partial_2\} - \nabla\{[\partial_2,\partial_1]\})\boldsymbol{\zeta} = \mathbf{K}\{\partial_1,\partial_2\}\cdot\boldsymbol{\zeta} - \boldsymbol{\zeta}\cdot\mathbf{K}\{\partial_1,\partial_2\}.$$

It remains to prove, that $\mathbf{K}\{\partial_1,\partial_2\}$ is a G-vector field:

$$\begin{aligned}&\partial_2\boldsymbol{\psi}_C(\boldsymbol{\Gamma}\{\partial_1\}) - \partial_1\boldsymbol{\psi}_C(\boldsymbol{\Gamma}\{\partial_2\}) + \boldsymbol{\psi}_C(\boldsymbol{\Gamma}\{\partial_2\})\cdot\boldsymbol{\psi}_C(\boldsymbol{\Gamma}\{\partial_1\})\\ &\quad - \boldsymbol{\psi}_C(\boldsymbol{\Gamma}\{\partial_1\})\cdot\boldsymbol{\psi}_C(\boldsymbol{\Gamma}\{\partial_2\}) - \boldsymbol{\psi}_C(\boldsymbol{\Gamma}\{[\partial_2,\partial_1]\})\\ &= (\partial_2\boldsymbol{\alpha})\cdot\boldsymbol{\Gamma}\{\partial_1\}\cdot\boldsymbol{\alpha}^{-1} + \boldsymbol{\alpha}\cdot(\partial_2\boldsymbol{\Gamma}\{\partial_1\})\cdot(\partial_2\boldsymbol{\alpha}^{-1}) - (\partial_2\partial_1\boldsymbol{\alpha})\cdot\boldsymbol{\alpha}^{-1} - (\partial_1\boldsymbol{\alpha})\cdot(\partial_2\boldsymbol{\alpha}^{-1})\\ &\quad - (\partial_1\boldsymbol{\alpha})\cdot\boldsymbol{\Gamma}\{\partial_2\}\cdot\boldsymbol{\alpha}^{-1} - \boldsymbol{\alpha}\cdot(\partial_1\boldsymbol{\Gamma}\{\partial_2\})\cdot\boldsymbol{\alpha}^{-1} - \boldsymbol{\alpha}\cdot\boldsymbol{\Gamma}\{\partial_2\}\cdot(\partial_1\boldsymbol{\alpha}^{-1})\\ &\quad + \boldsymbol{\alpha}\cdot\boldsymbol{\Gamma}\{\partial_2\}\cdot\boldsymbol{\Gamma}\{\partial_1\}\cdot\boldsymbol{\alpha}^{-1} + \boldsymbol{\alpha}\cdot\boldsymbol{\Gamma}\{\partial_1\}\cdot(\partial_2\boldsymbol{\alpha}^{-1}) + (\partial_1\boldsymbol{\alpha})\cdot\boldsymbol{\Gamma}\{\partial_2\}\cdot\boldsymbol{\alpha}^{-1}\\ &\quad + (\partial_1\boldsymbol{\alpha})\cdot(\partial_2\boldsymbol{\alpha}^{-1}) - \boldsymbol{\alpha}\cdot\boldsymbol{\Gamma}\{[\partial_2,\partial_1]\}\cdot\boldsymbol{\alpha}^{-1} + (\partial_2\partial_1\boldsymbol{\alpha})\cdot\boldsymbol{\alpha}^{-1}\\ &= \boldsymbol{\alpha}\cdot(\partial_2\boldsymbol{\Gamma}\{\partial_1\} - \partial_1\boldsymbol{\Gamma}\{\partial_2\} + \boldsymbol{\Gamma}\{\partial_2\}\cdot\boldsymbol{\Gamma}\{\partial_1\} - \boldsymbol{\Gamma}\{\partial_1\}\cdot\boldsymbol{\Gamma}\{\partial_2\} - \boldsymbol{\Gamma}\{[\partial_2,\partial_1]\})\cdot\boldsymbol{\alpha}^{-1}.\end{aligned}$$

In conclusion, if on a manifold $\mathbf{M}$ Riemannian metric is defined and if as an algebraic fibration over such manifold the fibration of Clifford algebras is given, then $\mathrm{Spin}(\mathbf{M})$ is such gauge group actions of which on vector and spinor fields preserve Riemannian metric. In this case G-connection for differential operators $\partial = \xi^k\partial/\partial x^k$ will be a Riemannian connection and G-vector field $\mathbf{K}\{\partial_1,\partial_2\}$ will be a tensor field of Riemannian curvature (see [5]).

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