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On Almost Generalized Weakly Symmetric Kenmotsu Manifolds

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Abstract

This paper aims to introduce the notions of an almost generalized weakly symmetric Kenmotsu manifolds and an almost generalized weakly Ricci-symmetric Kenmotsu manifolds. The existence of an almost generalized weakly symmetric Kenmotsu manifold is ensured by a non-trivial example.

Key words: Almost generalized weakly symmetric Kenmotsu manifolds, almost generalized weakly Ricci-symmetric Kenmotsu manifolds.

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1 Introduction

The notion of a weakly symmetric Riemannian manifold has been initiatied by Tamássy and Binh [22]. In the spirit of [5], a *weakly symmetric Riemannian manifold* (M^n, g), is said to be an n-dimensional almost weakly pseudo symmetric manifold $(n > 2)$, if its curvature tensor R of type $(0, 4)$ is not identically zero and admits the identity

$$\begin{aligned}(\nabla_X R)(Y,U,V,W) &= [A_1(X)+B_1(X)]\,R(Y,U,V,W)\\ &+ C_1(Y)\,R(X,U,V,W)+C_1(U)\,R(Y,X,V,W)\\ &+ D_1(V)\,R(Y,U,X,W)+D_1(W)\,R(Y,U,V,X), \end{aligned}\tag{1.1}$$

where A_1, B_1, C_1, D_1 are non-zero 1-forms defined by $A_1(X) = g(X,\sigma_1)$, $B_1(X) = g(X,\varrho_1)$, $C_1(X) = g(X,\pi_1)$ and $D_1(X) = g(X,\partial_1)$, for all X and $R(Y,U,V,W) = g(R(Y,U)V,W)$, ∇ being the operator of the covariant differentiation with respect to the metric tensor g. An n-dimensional Riemannian manifold of this kind is denoted by $A(WPS)_n$-manifold.

Keeping the tune of Dubey [8], we shall call a Riemannian manifold of dimension n an *almost generalized weakly symmetric* (which is abbreviated hereafter as $A(GWS)_n$-manifold) if it admits the equation

$$\begin{aligned}(\nabla_X R)(Y,U,V,W) &= [A_1(X)+B_1(X)]R(Y,U,V,W)+C_1(Y)R(X,U,V,W)\\ &+ C_1(U)R(Y,X,V,W)+D_1(V)R(Y,U,X,W)\\ &+ D_1(W)R(Y,U,V,X)+[A_2(X)+B_2(X)]\bar{G}(Y,U,V,W)\\ &+ C_2(Y)\bar{G}(X,U,V,W)+C_2(U)\bar{G}(Y,X,V,W)\\ &+ D_2(V)\;\bar{G}(Y,U,X,W)+D_2(W)\bar{G}(Y,U,V,X)\end{aligned}\tag{1.2}$$

where

$$G(Y,U,V,W) = g(U,V)g(Y,W) - g(Y,V)g(U,W)$$

and A_i, B_i, C_i, D_i, $i = 1,2$, are non-zero 1-forms defined by

$$A_i(X) = g(X,\sigma_i),\quad B_i(X) = g(X,\varrho_i),\quad C_i(X) = g(X,\pi_i),\quad D_i(X) = g(X,\partial_i).$$

The beauty of such $A(GWS)_n$-manifold is that it has the flavour of

(i) *locally symmetric space* in the sense of Cartan for $A_i = B_i = C_i = D_i = 0$,

(ii) *recurrent space* by Walker [24] for $A_1 \neq 0$, $B_i = C_i = D_i = 0$,

(iii) *generalized recurrent space* by Dubey[8] for $A_i \neq 0$ and $B_i = C_i = D_i = 0$,

(iv) *pseudo symmetric space* by Chaki [4] for $A_1 = B_1 = C_1 = D_1 \neq 0$ and $A_2 = B_2 = C_2 = D_2 = 0$,

(v) *semi-pseudo symmetric space* in the sense of Tarafder et al. [23] for $A_1 = -B_1$, $C_1 = D_1$ and $A_2 = B_2 = C_2 = D_2 = 0$,

(vi) *generalized semi-pseudo symmetric space* in the sense of Baishya [1] for $A_1 = -B_1$, $C_1 = D_1$ and $A_2 = -B_2$, $C_2 = D_2 = 0$,

(vii) *generalized pseudo symmetric space*, by Baishya [2] for $A_i = B_i = C_i = D_i \neq 0$,

(viii) *almost pseudo symmetric space* in the sprite of Chaki et al. [5] for $B_1 \neq 0$, $A_1 = C_1 = D_1 \neq 0$ and $A_2 = B_2 = C_2 = D_2 = 0$,

(ix) *almost generalized pseudo symmetric space* in the sence of Baishya for $B_i \neq 0$, $A_i = C_i = D_i \neq 0$,

(x) *weakly symmetric space* by Tamássy and Binh [22]
for $A_2 = B_2 = C_2 = D_2 = 0$.

Our work is structured as follows. Section 2 is concerned with Kenmotsu manifolds and some known results. In section 3, we have investigated an almost generalized weakly symmetric Kenmotsu manifold and obtained some interesting results. Section 4, is concerned with an almost generalized weakly Ricci-symmetric Kenmotsu manifold. Finally, we have constructed an example of an almost generalized weakly symmetric Kenmotsu manifold.

2 Kenmotsu manifolds and some known results

Let M be a n-dimensional connected differentiable manifold of class C^∞-covered by a system of coordinate neighborhoods (U, x^h) in which there are given a tensor field φ of type $(1,1)$, a cotravariant vector field ξ and a $1-$form η such that

$$\varphi^2 X = -X + \eta(X)\,\xi, \tag{2.1}$$

$$\eta(\xi) = 1, \quad \varphi \cdot \xi = 0, \quad \eta(\varphi X) = 0, \tag{2.2}$$

for any vector field X on M. Then the structure (φ, ξ, η) is called contact structure and the manifold M^n equipped with such structure is said to be an almost contact manifold, if there is given a Riemannian compatible metric g such that

$$g(\varphi X, Y) = -g(X, \varphi Y), \quad g(X, \xi) = \eta(X), \tag{2.3}$$

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.4}$$

for all vector fields X and Y, then we say M is an almost contact metric manifold.

An almost contact metric manifold M is called a *Kenmotsu manifold* if it satisfies [11]

$$(\nabla_X \varphi)Y = -g(X, \varphi Y)\xi - \eta(Y)\varphi(X), \tag{2.5}$$

for all vector fields X and Y, where ∇ is a Levi-Civita connection of the Riemannian metric. From the above it follows that

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.6}$$

$$(\nabla_X \eta)Y = g(X, Y) - \eta(X)\,\eta(Y), \tag{2.7}$$

In a Kenmotsu manifold the following relations hold ([7], [10])

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.8}$$

$$S(X, \xi) = -(n-1)\,\eta(X), \tag{2.9}$$

$$R(X, \xi)Y = g(X, Y)\xi - \eta(Y)X, \tag{2.10}$$

$$R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi \tag{2.11}$$

for any vector fields X, Y, Z, where R is the Riemannian curvature tensor of the manifold.

3 Almost generalized weakly symmetric Kenmotsu manifold

A Kenmotsu manifold (M^n, g) is said to be an almost generalized weakly symmetric if it admits the relation (1.1), $(n > 2)$.

Now, contracting Y over W in both sides of (1.1), we get

$$\begin{aligned}(\nabla_X S)(U,V) \;=\; & [A_1(X) + B_1(X)]S(U,V) + C_1(U)S(X,V) \\ & + C_1(R(X,U)V) + D_1(R(X,V)U) + D_1(V)S(U,X) \\ & + (n-1)[\{A_2(X) + B_2(X)\}g(U,V) + C_2(U)g(X,V) \\ & + D_2(V)g(U,X)] + C_2(G(X,U)V) + D_2(G(X,V)U). \qquad (3.1)\end{aligned}$$

In consequence of (2.8), (2.9) and (2.10) for $V = \xi$ the above equation yields

$$\begin{aligned}(\nabla_X S)(U,\xi) \;=\; & -(n-1)[A_1(X) + B_1(X)]\eta(U) - (n-2)C_1(U)\eta(X) \\ & + D_1(\xi)S(U,X) - \eta(U)\,C_1(X) - \eta(U)D_1(X) \\ & + g(X,U)D_1(\xi) + (n-1)[\{A_2(X) + B_2(X)\eta(U) \\ & + C_2(U)\eta(X) + D_2(\xi)g(U,X)] + \eta(U)C_2(X) \\ & - \eta(X)C_2(U) + \eta(U)D_2(X) - g(U,X)D_2(\xi). \qquad (3.2)\end{aligned}$$

Again, replacing V by ξ, in the following identity

$$(\nabla_X S)(U,V) = \nabla_X S(U,V) - S(\nabla_X U, V) - S(U, \nabla_X V) \qquad (3.3)$$

and then making use of (2.1), (2.6), (2.9), we find

$$(\nabla_X S)(U,\xi) = -(n-1)g(X,U) - S(U,X). \qquad (3.4)$$

Now, using (3.4) in (3.2), we have

$$\begin{aligned}& -(n-1)g(X,U) - S(U,X) = -(n-1)[\{A_1(X) + B_1(X)\}\eta(U)] \\ & \quad - (n-2)C_1(U)\eta(X) + D_1(\xi)S(U,X) \\ & \quad - \eta(U)C_1(X) + g(X,U)D_1(\xi) - \eta(U)D_1(X) \\ & \quad + (n-1)[\{A_2(X) + B_2(X)\}\eta(U) + C_2(U)\eta(X) + D_2(\xi)g(U,X)] \\ & \quad + \eta(U)C_2(X) - \eta(X)C_2(U) + \eta(U)D_2(X) - g(U,X)D_2(\xi) \qquad (3.5)\end{aligned}$$

which leaves

$$[A_1(\xi) + B_1(\xi) + C_1(\xi) + D_1(\xi)] = [A_2(\xi) + B_2(\xi) + C_2(\xi) + D_2(\xi)] \quad (3.6)$$

for $X = U = \xi$.

In particular, if $A_2(\xi) = B_2(\xi) = C_2(\xi) = D_2(\xi) = 0$, formula (3.6) turns into

$$A_1(\xi) + B_1(\xi) + C_1(\xi) + D_1(\xi) = 0. \qquad (3.7)$$

This leads to the following

Theorem 1. *In an almost generalized weakly symmetric Kenmotsu manifold* (M^n, g), $n > 2$, *the relation* (3.6) *hold good.*

In a similar manner, we can have

$$\begin{aligned} &-(n-1)g(X,V) - S(V,X) \\ &= -(n-1)[A_1(X) + B_1(X)]\eta(V) - (n-2)D_1(V)\eta(X)] \\ &\quad + C_1(\xi)S(X,V) + g(X,V)C_1(\xi) - \eta(V)C_1(X) - \eta(V)D_1(X) \\ &\quad + (n-1)[\{A_2(X) + B_2(X)\}\eta(V) + C_2(\xi)g(X,V) \\ &\quad + D_2(V)\eta(X)] + \eta(V)C_2(X) - g(X,V)C_2(\xi) \\ &\quad + \eta(V)D_2(X) - \eta(X)D_2(V) \end{aligned} \tag{3.8}$$

Now, putting $V = \xi$ in (3.8) and using (2.1), (2.9), we obtain

$$\begin{aligned} &(n-1)[A_1(X) + B_1(X)] + C_1(X) + D_1(X) \\ &\qquad + (n-2)[C_1(\xi) + D_1(\xi)]\eta(X) \\ &\quad = [(n-1)\{A_2(X) + B_2(X)\} + (n-2)\{C_2(\xi) + D_2(\xi)\}\eta(X)] \\ &\qquad + C_2(X) + D_2(X) \end{aligned} \tag{3.9}$$

Putting $X = \xi$ in (3.8) and using (2.1), (2.2), (2.9), we obtain

$$\begin{aligned} &(n-1)[A_1(\xi) + B_1(\xi) + C_1(\xi)]\eta(V) + (n-2)D_1(V) + \eta(V)D_1(\xi) \\ &\quad = (n-1)[\{A_2(\xi) + B_2(\xi) + C_2(\xi)\}\eta(V) + D_2(V)] \\ &\qquad + \eta(V)D_2(\xi) - D_2(V) \end{aligned} \tag{3.10}$$

Replacing V by X in the above equation and using (3.6), we get

$$D_1(X) - D_1(\xi)\eta(X) = D_2(X) - D_2(\xi)\eta(X) \tag{3.11}$$

In view of (3.6), (3.9) and (3.11), we get

$$C_1(X) - C_1(\xi)\eta(X) = C_2(X) - C_2(\xi)\eta(X). \tag{3.12}$$

Subtracting (3.11), (3.12) from (3.9), we get

$$\begin{aligned} &[A_1(X) + B_1(X)] + [C_1(\xi) + D_1(\xi)]\eta(X) \\ &\qquad = \{A_2(X) + B_2(X)\} + \{C_2(\xi) + D_2(\xi)\}\eta(X)] \end{aligned} \tag{3.13}$$

Again, adding (3.11), (3.12) and (3.13), we get

$$\begin{aligned} &A_1(X) + B_1(X) + C_1(X) + D_1(X) \\ &\qquad = [A_2(X) + B_2(X) + C_2(X) + D_2(X)]. \end{aligned} \tag{3.14}$$

Next, in view of $A_2 = B_2 = C_2 = D_2 = 0$, the relation (3.14) yields

$$A_1(X) + B_1(X) + C_1(X) + D_1(X) = 0. \tag{3.15}$$

This motivates us to state the followings

Theorem 2. *In an almost generalized weakly symmetric Kenmotsu manifold* (M^n, g), $n > 3$, *the sum of the associated 1-forms is given by* (3.14).

Theorem 3. *There does not exist a Kenmotsu manifold which is*

(i) recurrent,

(ii) generalized recurrent provided the 1-forms are collinear,

(iii) pseudo symmetric,

(iv) generalized semi-pseudo symmetric provided the 1-forms are collinear,

(v) generalized almost-pseudo symmetric provided the 1-forms are collinear.

4 Almost generalized weakly Ricci-symmetric Kenmotsu manifold

A Kenmotsumanifold $(M^n, g)(n > 3)$, is said to be almost generalized weakly Ricci-symmetric if there exist 1-forms $\bar{A}_i$, $\bar{B}_i$, $\bar{C}_i$ and $\bar{D}_i$ which satisfy the condition

$$\begin{aligned}&(\nabla_X S)(U,V)\\&\quad= [\bar{A}_1(X) + \bar{B}_1(X)]S(U,V) + \bar{C}_1(U)S(X,V) + \bar{D}_1(V)S(U,X)\\&\qquad+ [\bar{A}_2(X) + \bar{B}_2(X)]g(U,V) + \bar{C}_2(U)g(X,V) + \bar{D}_2(V)g(U,X).\end{aligned}\tag{4.1}$$

Putting $V = \xi$ in (4.1), we obtain

$$\begin{aligned}&(\nabla_X S)(U,\xi)\\&= [\bar{A}_1(X) + \bar{B}_1(X)](n-2)\eta(U) + \bar{C}_1(U)(n-2)\eta(X) + \bar{D}_1(\xi)S(U,X)\\&\qquad+ [\bar{A}_2(X) + \bar{B}_2(X)]\eta(U) + \bar{C}_2(U)\eta(X) + \bar{D}_2(\xi)g(U,X).\end{aligned}\tag{4.2}$$

In view of (3.4), the relation (4.2) becomes

$$\begin{aligned}&-(n-1)g(X,U) - S(U,X)\\&\quad= -(n-1)[\{\bar{A}_1(X) + \bar{B}_1(X)\}\eta(U) + \bar{C}_1(U)\eta(X)] + \bar{D}_1(\xi)S(U,X)\\&\qquad+ [\bar{A}_2(X) + \bar{B}_2(X)]\eta(U) + \bar{C}_2(U)\eta(X) + \bar{D}_2(\xi)g(U,X).\end{aligned}\tag{4.3}$$

Setting $X = U = \xi$ in (4.3) and using (2.1), (2.2) and (2.9), we get

$$\begin{aligned}&(n-1)[\bar{A}_1(\xi) + \bar{B}_1(\xi) + \bar{C}_1(\xi) + \bar{D}_1(\xi)]\\&\qquad= [\bar{A}_2(\xi) + \bar{B}_2(\xi) + \bar{C}_2(\xi) + \bar{D}_2(\xi)].\end{aligned}\tag{4.4}$$

Again, putting $X = \xi$ in (4.3), we get

$$\begin{aligned}&(n-1)[\{\bar{A}_1(\xi) + \bar{B}_1(\xi) + \bar{D}_1(\xi)\}\eta(U) + \bar{C}_1(U)]\\&\qquad= [\bar{A}_2(\xi) + \bar{B}_2(\xi) + \bar{D}_2(\xi)]\eta(U) + \bar{C}_2(U).\end{aligned}\tag{4.5}$$

Setting $U = \xi$ in (4.3) and then using (2.1), (2.2) and (2.9), we obtain

$$\begin{aligned}&(n-1)[\{\bar{A}_1(X) + \bar{B}_1(X)\} + \{\bar{C}_1(\xi) + \bar{D}_1(\xi)\}\eta(X)]\\&\qquad= [\bar{A}_2(X) + \bar{B}_2(X)] + \bar{C}_2(\xi)\eta(X) + \bar{D}_2(\xi)\eta(X).\end{aligned}\tag{4.6}$$

Replacing U by X in (4.5) and adding with (4.6), we have

$$\begin{aligned}(n-1)[\bar{A}_1(X)+\bar{B}_1(X)+\bar{C}_1(X)]-[\bar{A}_2(X)+\bar{B}_2(X)+\bar{C}_2(X)]\\ =-(n-1)[\bar{A}_1(\xi)+\bar{B}_1(\xi)+\bar{C}_1(\xi)+\bar{D}_1(\xi)]\eta(X)\\ +[\bar{A}_2(\xi)+\bar{B}_2(\xi)+\bar{C}_2(\xi)+\bar{D}_2(\xi)]\eta(X)\\ -(n-1)\bar{D}_1(\xi)\eta(X)+\bar{D}_2(\xi)\eta(X).\end{aligned} \tag{4.7}$$

In consequence of (4.4), the above equation becomes

$$\begin{aligned}(n-1)[\bar{A}_1(X)+\bar{B}_1(X)+\bar{C}_1(X)]+(n-1)\bar{D}_1(\xi)\eta(X).\\ =[\bar{A}_2(X)+\bar{B}_2(X)+\bar{C}_2(X)]+\bar{D}_2(\xi)\eta(X)\end{aligned} \tag{4.8}$$

Next, putting $X=U=\xi$ in (4.1), we get

$$\begin{aligned}(n-1)[\bar{A}_1(\xi)+\bar{B}_1(\xi)+\bar{C}_1(\xi)]\eta(V)+(n-1)\bar{D}_1(V)\\ =[\bar{A}_2(\xi)+\bar{B}_2(\xi)+\bar{C}_2(\xi)]\eta(V)+\bar{D}_2(V)\end{aligned} \tag{4.9}$$

Replacing V by X in (4.9) and adding with (4.8), we obtain

$$\begin{aligned}(n-1)[\bar{A}_1(X)+\bar{B}_1(X)+\bar{C}_1(X)+\bar{D}_1(X)]\\ (n-1)[\bar{A}_1(\xi)+\bar{B}_1(\xi)+\bar{C}_1(\xi)+\bar{D}_1(\xi)]\eta(V)\\ =[\bar{A}_2(X)+\bar{B}_2(X)+\bar{C}_2(X)+\bar{D}_1(X)]\\ +[\bar{A}_2(\xi)+\bar{B}_2(\xi)+\bar{C}_2(\xi)+\bar{D}_2(\xi)]\eta(V).\end{aligned} \tag{4.10}$$

By virtue of (4.4), the above equation becomes

$$\begin{aligned}(n-1)[\bar{A}_1(X)+\bar{B}_1(X)+\bar{C}_1(X)+\bar{D}_1(X)]\\ =[\bar{A}_2(X)+\bar{B}_2(X)+\bar{C}_2(X)+\bar{D}_1(X)].\end{aligned} \tag{4.11}$$

This leads to the followings

Theorem 4. *In an almost generalized weakly Ricci symmetric Kenmotsu manifold*

Theorem 5. *(M^n, g), $n>2$, the sum of the associated 1-forms are related by* (4.11).

5 Example of an A(GWS)$_3$ Kenmotsu manifold

(see [7], page 21–22) Let $M^3(\varphi,\xi,\eta,g)$ be a Kenmotsu manifold (M^3,g) with a φ-basis

$$e_1=e^{-z}\frac{\partial}{\partial x},\quad e_2=e^{-z}\frac{\partial}{\partial y},\quad e_3=\frac{\partial}{\partial z}.$$

Then from Koszul's formula for Riemannian metric g, we can obtain the Levi-Civita connection as follows

$$\begin{aligned}
\nabla_{e_1} e_3 &= e_1, \quad \nabla_{e_1} e_2 = 0, \quad \nabla_{e_1} e_1 = -e_3,\\
\nabla_{e_2} e_3 &= e_2, \quad \nabla_{e_2} e_2 = -e_3, \quad \nabla_{e_2} e_1 = 0,\\
\nabla_{e_3} e_3 &= 0, \quad \nabla_{e_3} e_2 = 0, \quad \nabla_{e_3} e_1 = 0.
\end{aligned}$$

Using the above relations, one can easily calculate the non-vanishing components of the curvature tensor R (up to symmetry and skew-symmetry)

$$R(e_1, e_3, e_1, e_3) = R(e_2, e_3, e_2, e_3) = 1 = R(e_1, e_2, e_1, e_2).$$

Since $\{e_1, e_2, e_3\}$ forms a basis, any vector field $X, Y, U, V \in \chi(M)$ can be written as

$$X = \sum_1^3 a_i e_i, \quad Y = \sum_1^3 b_i e_i, \quad U = \sum_1^3 c_i e_i, \quad V = \sum_1^3 d_i e_i,$$

$$\begin{aligned}
R(X, Y, U, V) &= (a_1 b_2 - a_2 b_1)(c_1 d_2 - c_2 d_1) + (a_1 b_3 - a_3 b_1)(c_1 d_3\\
&\quad - c_3 d_1) + (a_2 b_3 - a_3 b_2)(c_2 d_3 - c_3 d_2) = T_1 \text{ (say)}\\
R(e_1, Y, U, V) &= b_3(c_1 d_3 - c_3 d_1) + b_2(c_1 d_2 - c_2 d_1) = \lambda_1 \text{ (say)}\\
R(e_2, Y, U, V) &= b_3(c_2 d_3 - c_3 d_2) - b_1(c_1 d_2 - c_2 d_1) = \lambda_2 \text{ (say)}\\
R(e_3, Y, U, V) &= b_1(c_3 d_1 {}_- c_1 d_3) + b_2(c_3 d_2 - c_2 d_3) = \lambda_3 \text{ (say)}\\
R(X, e_1, U, V) &= a_3(c_1 d_3 - c_3 d_1) + a_2(c_1 d_2 - c_2 d_1) = \lambda_4 \text{ (say)}\\
R(X, e_2, U, V) &= a_3(c_2 d_3 - c_3 d_2) + a_1(c_2 d_1 - c_1 d_2) = \lambda_5 \text{ (say)}\\
R(X, e_3, U, V) &= a_1(c_3 d_1 {}_- c_1 d_3) + a_2(c_3 d_2 - c_2 d_3) = \lambda_6 \text{ (say)}\\
R(X, Y, e_1, V) &= d_3(a_1 b_3 - a_3 b_1) + d_2(a_1 b_2 - a_2 b_1)(= \lambda_7 \text{ (say)}\\
R(X, Y, e_2, V) &= d_3(a_2 b_3 - a_3 b_2) + d_1(a_2 b_1 - a_1 b_2) = \lambda_8 \text{ (say)}\\
R(X, Y, e_3, V) &= d_1(a_3 b_1 - a_1 b_3\ ) + d_2(a_3 b_2 - a_2 b_3) = \lambda_9 \text{ (say)}\\
R(X, Y, U, e_1) &= c_3(a_1 b_3 - a_3 b_1) + c_2(a_1 b_2 - a_2 b_1) = \lambda_{10} \text{ (say)}\\
R(X, Y, U, e_2) &= c_3(a_2 b_3 - a_3 b_2\ ) + c_1(a_2 b_1 - a_1 b_2) = \lambda_{11} \text{ (say)}\\
R(X, Y, U, e_3) &= c_1(a_3 b_1 - a_1 b_3) + c_2(a_3 b_2 - a_2 b_3) = \lambda_{12} \text{ (say)}\\
G(X, Y, U, V) &= (b_1 c_1 + b_2 c_2 - b_3 c_3)(a_1 d_1 + a_2 d_2 - a_3 d_3)\\
&\quad - (a_1 c_1 + a_2 c_2 - a_3 c_3)(b_1 d_1 + b_2 d_2 - b_3 d_3) = T_2 \text{ (say)}\\
G(e_1, Y, U, V) &= (b_2 c_2 - b_3 c_3) d_1 - (b_2 d_2 - b_3 d_3) c_1 = \omega_1 \text{ (say)}\\
G(e_2, Y, U, V) &= (b_1 c_1 - b_3 c_3) d_2 - (b_1 d_1 - b_3 d_3) c_2 = \omega_2 \text{ (say)}\\
G(e_3, Y, U, V) &= (b_1 c_1 - b_2 c_2) d_3 - (b_1 d_1 - b_2 d_2) c_3 = \omega_3 \text{ (say)}\\
G(X, e_1, U, V) &= (a_2 d_2 - a_3 d_3) c_1 - (a_2 c_2 - a_3 c_3) d_1 = \omega_4 \text{ (say)}\\
G(X, e_2, U, V) &= (a_1 d_1 - a_3 d_3) c_2 - (a_1 c_1 - a_3 c_3) d_2 = \omega_5 \text{ (say)}\\
G(X, e_3, U, V) &= (a_1 d_1 - a_2 d_2) c_3 - (a_1 c_1 - a_2 c_2) d_3 = \omega_6 \text{ (say)}
\end{aligned}$$

$$
\begin{aligned}
G(X,Y,e_1,V) &= (a_2d_2 - a_3d_3)b_1 - (b_2d_2 - b_3d_3)a_1 = \omega_7 \text{ (say)}\\
G(X,Y,e_2,V) &= (a_1d_1 - a_3d_3)b_2 - (b_1d_1 - b_3d_3)a_2 = \omega_8 \text{ (say)}\\
G(X,Y,e_3,V) &= (b_1d_1 - b_2d_2)a_3 - (a_1d_1 - a_2d_2)b_3 = \omega_9 \text{ (say)}\\
G(X,Y,U,e_1) &= (b_2c_2 - b_3c_3)a_1 - (a_2c_2 - a_3c_3)b_1, = \omega_{10} \text{ (say)}\\
G(X,Y,U,e_2) &= (b_1c_1 - b_3c_3)a_2 - (a_1c_1 - a_3c_3)b_2 = \omega_{11} \text{ (say)}\\
G(X,Y,U,e_3) &= (b_1c_1 - -b_2c_2)a_3 - (a_1c_1 + a_2c_2)b_3 = \omega_{12} \text{ (say)}
\end{aligned}
$$

and the components which can be obtained from these by the symmetry properties. Now, we calculate the covariant derivatives of the non-vanishing components of the curvature tensor as follows

$$
\begin{aligned}
&(\nabla_{e_1}R)(X,Y,U,V) =\\
&\quad = -a_1\lambda_3 + a_3\lambda_2 - b_1\lambda_6 + b_3\lambda_5 - c_1\lambda_9 + c_3\lambda_8 - d_1\lambda_{12} + d_3\lambda_{11},\\
&(\nabla_{e_2}R)(X,Y,U,V) =\\
&\qquad = -a_2\lambda_3 + a_3\lambda_2 - b_2\lambda_6 + b_3\lambda_5 - c_2\lambda_9 + c_3\lambda_8 d_3\lambda_{11} - d_2\lambda_{12},\\
&(\nabla_{e_3}R)(X,Y,U,V) = 0,
\end{aligned}
$$

For the following choice of the the one forms

$$
\begin{aligned}
A_1(e_1) &= \frac{a_3\lambda_2 - a_1\lambda_3}{T_1}, \qquad B_1(e_1) = \frac{b_3\lambda_5 - b_1\lambda_6}{T_1},\\
A_2(e_1) &= \frac{c_3\lambda_8 - c_1\lambda_9}{T_2}, \qquad B_2(e_1) = \frac{d_3\lambda_{11} - d_1\lambda_{12}}{T_2},\\
A_1(e_2) &= \frac{a_3\lambda_2 - a_2\lambda_3}{T_1}, \qquad B_1(e_2) = \frac{b_3\lambda_5 - b_2\lambda_6}{T_1},\\
A_2(e_2) &= \frac{c_3\lambda_8 - c_2\lambda_9}{T_2}, \qquad B_2(e_2) = \frac{d_3\lambda_{11} - d_2\lambda_{12}}{T_2},\\
A_1(e_3) &= \frac{e^{2z}(a_1b_2 - a_2b_1)c_1d_2}{T_1},\\
B_1(e_3) &= -\frac{e^{2z}(a_1b_2 - a_2b_1)c_1d_2}{T_1},\\
C_1(e_3) &= \frac{1}{a_3\lambda_3 + b_3\lambda_6}, \qquad C_2(e_3) = \frac{1}{a_3\theta_3 + b_3\theta_6},\\
D_1(e_3) &= -\frac{1}{c_3\lambda_9 + d_3\lambda_{12}}, \qquad D_2(e_3) = -\frac{1}{c_3\theta_9 + d_3\theta_{12}},\\
A_2(e_3) &= \frac{\alpha^2e^{2z}(a_1b_2 - a_2b_1)c_1d_2}{T_2},\\
B_2(e_3) &= -\frac{\alpha^2e^{2z}(a_1b_2 - a_2b_1)c_2d_1}{T_2},
\end{aligned}
$$

one can easily verify the relations

$$\begin{aligned}(\nabla_{e_i} R)(X,Y,U,V) = {} & [A_1(e_i)+B_1(e_i)]R(X,Y,U,V) \\ & + C_1(X)R(e_i,Y,U,V) + C_1(Y)R(X,e_i,U,V) \\ & + D_1(U)R(X,Y,e_i,V) + D_1(V)R(X,Y,U,e_i) \\ & + [A_2(e_i)+B_2(e_i)]G(X,Y,U,V) \\ & + C_2(X)G(e_i,Y,U,V) + C_2(Y)G(X,e_i,U,V) \\ & + D_2(U)G(X,Y,e_i,V) + D_2(V)G(X,Y,U,e_i)\end{aligned}$$

for 1, 2, 3. From the above, we can state that

Theorem 6. *There exixt a Kenmotsu manifold* (M^3, g) *which is an almost generalized weakly symmetry Kenmotsu manifold.*

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