Integral Presentations of Deviations of
de la Vallee Poussin Right-Angled Sums

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Abstract
We investigate approximation properties of de la Vallee Poussin right-
angled sums on the classes of periodic functions of several variables with
a high smoothness. We obtain integral presentations of deviations of de
la Vallee Poussin sums on the classes $C_{\beta,\infty}^{m\alpha}$. 

Key words: Right-angled sums of Vallee Poussin, integral presenta-
tions, Fourier series.

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1 Introduction
Considering [1] we define $\bar{\psi}$-integral classes of periodic functions of several vari-
bles in the following way.

Let $R^m$ be an Euclidean space with elements $\bar{x} = (x_1, x_2, \ldots, x_m)$, and let
$T^m = \prod_{i=1}^{m}[-\pi; \pi]$ be an m-dimensional cube with the side $2\pi$,

$N^m = \{ \bar{x} \in R^m \mid x_i \in N, \ i = 1, 2, \ldots, m \}$,

$N^*_m = \{ \bar{x} \in R^m \mid x_i \in N_\ast = N \cup \{0\}, \ i = 1, 2, \ldots, m \}$,

$N^m_i = \{ \bar{x} \in R^m \mid x_i \in N, x_j \in N_\ast, \ i \neq j \}$,

$E^m = \{ \bar{x} \in R^m \mid x_i \in \{0; 1\}, \ i = 1, 2 \}$. 

We denote by $L(T^m)$ the set of summable on a cube $T^m$ functions $f(\bar{x}) = f(x_1, x_2, \ldots, x_m)$ which are $2\pi$-periodic on every variable.
Let $f \in L(T^m)$. Then for every pair of points $\vec{s} \in E^m$, $\vec{k} \in N_+^m$ we have a corresponding value

$$a_{\vec{k}}^\vec{s}(f) = \frac{1}{\pi^m} \int_{T^m} f(\vec{x}) \prod_{i=1}^m \cos \left( k_i x_i - \frac{s_i \pi}{2} \right) dx_i.$$

Values $a_{\vec{k}}^\vec{s}(f)$, $\vec{s} \in E^m$, $\vec{k} \in N_+^m$ are the Fourier coefficients of the function $f(\vec{x})$ [1, p. 546].

For every vector $\vec{k} \in N_+^m$ we have the major harmonic of the function $f(\vec{x})$

$$A_\vec{k}(f; \vec{x}) = \sum_{\vec{s} \in E^m} a_{\vec{k}}^\vec{s}(f) \prod_{i=1}^m \cos \left( k_i x_i - \frac{s_i \pi}{2} \right) \cos \left( \frac{(s_i + 1) \pi}{2} \right).$$

Using [1, p. 545] we define Fourier series of the function $f(\vec{x})$ by the following relation

$$S[f] = \sum_{\vec{k} \in N_+^m} \frac{1}{2q(\vec{k})} A_\vec{k}(f, \vec{x}),$$

where $q(\vec{k})$ is a number of zero coordinates of the vector $\vec{k}$.

Let $f \in L(T^m)$ and systems of numbers $\psi_{ij}(k)$, $\Psi_{ij}(k)$, $i = 1, 2, \ldots, m$; $j = 1, 2$, $k \in N_+$ be given.

Let us put

$$\overline{\psi}_i(k) = \sqrt{\psi_{1i}^2(k) + \psi_{2i}^2(k)}, \overline{\Psi}_i(k) = \sqrt{\Psi_{1i}^2(k) + \Psi_{2i}^2(k)}$$

and consider the following conditions be fulfilled: $\overline{\psi}_i(k) \neq 0$, $\overline{\Psi}_i(k) \neq 0$, $k \in N_+$, $\psi_{i1}(0) = 1$, $\Psi_{i1}(0) = 1$, $\psi_{i2}(0) = 0$, $\Psi_{i2}(0) = 0$, $i = 1, 2, \ldots, m$.

Furthermore, let

$$\sum_{\vec{k} \in N_+^m} \frac{1}{2q(\vec{k})\psi_i(k_i)} [\psi_{i1}(k_i) A_\vec{k}(f, \vec{x}) - \psi_{i2}(k_i) A_{\vec{k}}^\psi_i(f, \vec{x})]$$

be the Fourier series of some function of $L(T^m)$. It will be denoted by

$$f^{\overline{\psi}_i}(\vec{x}) = \frac{\partial f(\vec{x})}{\partial x_i}$$

and called $\overline{\psi}_i$-derivative of the function $f$ with respect to the $x_i$, $i \in m$. 

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Integral presentations of deviations of de la Vallee Poussin.

Let $\overline{m} = \{1, 2, \ldots, m\}$. For a fixed $r$-elemental set $\mu(r) \subset \overline{m}$, $\mu(r) = \{i_1, i_2, \ldots, i_r\}$, we define a function $f^{\Psi_{\mu}}(\overline{x})$ by

$$f^{\Psi_{\mu}}(\overline{x}) = \frac{\partial^r \Psi_{\mu r-1} \ldots \partial \Psi_{1} f(\overline{x})}{\partial x_{i_1} \partial x_{i_{r-1}} \ldots \partial x_{i_1}}$$

and call it mixed $\Psi_{\mu}$-derivative with respect to variables $x_i, i \in \mu(r)$.

Let a set of functions $\psi_{i_j}, \Psi_{i_j}, i = 1, 2, \ldots, m; j = 1, 2$ be given. The set of continuous functions $f \in L(T^m)$ having the essentially bounded $\Psi_{\mu}$- and $\Psi_1$-derivatives, i.e.

$$\text{ess sup } |f^{\Psi_{\mu}}(\overline{x})| \leq 1, \quad \text{ess sup } |f^{\Psi_1}(\overline{x})| \leq 1, \quad i = 1, 2, \ldots, m; \quad \mu \subset \overline{m}; \quad \overline{x} \in T^m$$

will be denoted by the symbol $C_{m\infty}^{\Psi_\infty}$.

If for the sets of functions $\psi_{i_j}(k)$ and $\Psi_{i_j}(k), i = 1, 2, \ldots, m; j = 1, 2$, the functions $\psi_i(k), \Psi_i(k)$ and numbers $\beta_i, \beta_1, i = 1, 2, \ldots, m$, fulfill

$$\psi_{11}(k) = \psi_1(k) \cos \frac{\beta_1 \pi}{2}; \quad \psi_{12}(k) = \psi_1(k) \sin \frac{\beta_1 \pi}{2};$$

then the class $C_{m\infty}^{\Psi_\infty}$ is the class of $(\psi, \beta)$-differentiable periodic functions of $m$ variables (see [2]) and it is denoted by $C_{m\infty}^{\Psi_\infty}$.

Let us put $C_{m\infty}^{\Psi_\infty} = \prod_{i=1}^{m} [n_i - p_i; n_i - 1]$ for $\vec{n} \in N^m, \vec{p} \in N^m, p_i < n_i, i = 1, 2, \ldots, m$. Then trigonometric polynomials of the type

$$V_{\vec{r}, \vec{p}}(f; \overline{x}) = \prod_{i=1}^{m} p_i \sum_{\vec{k} \in G_{\vec{r}, \vec{p}}} S_{\vec{k}}(f; \overline{x}),$$

(see [2]) are partial sums of Fourier right-angled sums.

In this work the problems of approximation of classes $C_{m\infty}^{\Psi_\infty}$ by polynomials $V_{\vec{r}, \vec{p}}(f; \overline{x})$ are investigated. The functions which determine these classes are defined in the following way:

$$\psi_i(x) = e^{-\alpha_i x}, \quad \Psi_i(x) = e^{-\alpha_i^* x}, \quad \alpha_i > 0, \quad \alpha_i^* > 0, \quad i = 1, 2, \ldots, m.$$
We denote such classes by $C_{\beta, \infty}^{m, \alpha}$ (analogously to the classes of functions of a single variable).

It is proved by S. M. Nikol’skii in [6] (see also [7], [8]) that for upper bounds of the deviations of Fourier sums on the corresponding classes $C_{\beta, \infty}^{m, \alpha}$ functions of one variable we obtain the following asymptotic equality for $n \to \infty$:

$$E \left( C_{\beta, \infty}^{m, \alpha}; S_n \right) = \frac{8q^n}{\pi^2} K(q) + O(1), \quad q = e^{-\alpha}, \quad (7)$$

where

$$K(q) = \int_0^{\frac{\pi}{2}} \frac{du}{\sqrt{1 - q^2 \sin^2 u}}$$

is the total elliptic integral of the first kind.

Asymptotic equalities for upper bounds of the deviations of de la Vallee Poussin sums on the classes $C_{\beta, \infty}^{m, \alpha}$ may be found in the [9], [10] (see also [11], [12, p. 217]):

$$E \left( C_{\beta, \infty}^{m, \alpha}; V_{n,p} \right) = \frac{4q^{n-p+1}}{\pi p(1 - q^2)} + O(1) \left( \frac{q^{n-p+1}}{n(n-p)(1-q^2)} + \frac{q^n}{p(1-q^2)} \right), \quad 1 < p < n. \quad (8)$$

The 2-dimensional and $m$-dimensional analogies of equality (7) for the classes $C_{\beta, \infty}^{m, \alpha}$ are in the works [13], [14].

2 Main Results

Let $\Lambda = \{ \Lambda_1, \Lambda_2, \ldots, \Lambda_m \}$ be a fixed set of infinite triangle numeric matrices, $\Lambda_i = \{ \lambda^{(n)}_{k_i}, i = 1, 2, \ldots, m, \lambda^{(0)}_{k_i} = 1, \lambda^{(n)}_{k_i} = 0 \}$ for $k_i \geq n_i$.

Further let $\lambda^{(n)}_k = \prod_{i=1}^m \lambda^{(n)}_{k_i}$ and let $G_{\vec{n}} = \prod_{i=1}^m [0; n_i - 1]$ be an right-angled parallelepiped corresponding to the vector $\vec{n} \in N^m$.

For every function with Fourier series (1) we have a trigonometric polynomial

$$U_{\vec{n}}(f; \vec{x}; \Lambda) = \sum_{\vec{k} \in G_{\vec{n}}} 2^{-n(\vec{k})} \lambda^{(n)}_k A_{\vec{k}}(f; \vec{x}).$$

Values $\delta_{\vec{n}}(f; \vec{x}; \Lambda) = f(\vec{x}) - U_{\vec{n}}(f; \vec{x}; \Lambda)$ are the deviations of such polynomials of the function $f(\vec{x})$.

In this work there are found the integral presentations of the deviations

$$\delta_{\vec{n}, \vec{p}}(f, \vec{x}) = f(\vec{x}) - V_{\vec{n}, \vec{p}}(f, \vec{x})$$

of sums $V_{\vec{n}, \vec{p}}(f, \vec{x})$ from function $f(\vec{x})$ out of classes $C_{\beta, \infty}^{m, \alpha}$.

The following theorem is the main result of this work.
Theorem 1 If $\alpha_i > 0$, $\alpha_i^* > 0$, $q_i = e^{-\alpha_i}$, $Q_i = e^{-\alpha_i^*}$, $\beta_i \in \mathbb{R}$, $\beta_i^* \in \mathbb{R}$, $p_i \in \mathbb{N}$, $1 < p_i < n_i$; $i = 1, 2, \ldots, m$,
then for every function $f \in \mathcal{C}_\beta^{m, \alpha}$ the following equality is fulfilled

$$
\delta_{\bar{n}, \bar{p}}(f; \bar{x}) = \sum_{i=1}^{m} \frac{q_i^{n_i-p_i+1}}{p_i \pi} \int_{-\pi}^{\pi} f_{\beta_i}^\psi(\bar{x} + t_i \bar{e}_i) b_{n_i-p_i}(t_i) dt_i
$$

$$
- \sum_{i=1}^{m} \frac{q_i^{n_i+1}}{p_i \pi} \int_{-\pi}^{\pi} f_{\beta_i}^\psi(\bar{x} + t_i \bar{e}_i) b_{n_i}(t_i) dt_i
$$

$$
+ O(1) \sum_{r=2}^{m} \sum_{\mu(r) \in \mathbb{N}} \prod_{j \in \mu(r)} Q_{r_j}^{n_j-p_j+1} \int_{T_r} \left| B_{s_j-p_j}(t_j) \right| dt_j,
$$

(9)

where

$$
b_{n_i}(t_i) = \left( \frac{q_i^2 \cos t_i - 2q_i \cos t_i + \cos (\alpha_i t_i)}{(1 - 2q_i \cos t_i + q_i^2)^2} \cos \left( n_i t_i + \frac{\beta_i \pi}{2} \right) \right)
$$

$$
+ \left( \frac{q_i^2 \sin t_i - \sin t_i}{(1 - 2q_i \cos t_i + q_i^2)^2} \sin \left( n_i t_i + \frac{\beta_i \pi}{2} \right) \right),
$$

$$
B_{s_j}(t_j) = \left( \frac{Q_{s_j}^2 \cos t_j - 2Q_{s_j} \cos t_j + \cos (\alpha_{s_j} t_j)}{(1 - 2Q_{s_j} \cos t_j + Q_{s_j}^2)^2} \cos \left( n_j t_j + \frac{\beta_{s_j} \pi}{2} \right) \right)
$$

$$
+ \left( \frac{Q_{s_j}^2 \sin t_j - \sin t_j}{(1 - 2Q_{s_j} \cos t_j + Q_{s_j}^2)^2} \sin \left( n_j t_j + \frac{\beta_{s_j} \pi}{2} \right) \right).
$$

Proof It is clear that

$$
\delta_{\bar{n}, \bar{p}}(f; \bar{x}) = \frac{1}{1 \prod_{i=1}^{m} p_i} \sum_{\bar{k} \in G_{\alpha, \beta}} \rho_{\bar{k}}(f; \bar{x}) = \frac{1}{1 \prod_{i=1}^{m} p_i} \sum_{i=1}^{m} \sum_{k_i = n_i-p_i}^{n_i-1} \rho_{\bar{k}}(f; \bar{x}),
$$

(10)

where

$$
\rho_{\bar{k}}(f; \bar{x}) = f(\bar{x}) - S_{\bar{k}}(f; \bar{x}), \quad \bar{k} = (k_1; k_2; \ldots; k_m).
$$

Let us investigate $\rho_{\bar{k}}(f; \bar{x})$. Using theorem 1 in [13] for $f \in \mathcal{C}_\beta^{m, \alpha}$ we have

$$
\rho_{\bar{k}}(f; \bar{x}) = \sum_{i=1}^{m} \frac{1}{\pi} \int_{-\pi}^{\pi} f_{\beta_i}^\psi(\bar{x} + t_i \bar{e}_i) \sum_{k=n_i+1}^{\infty} \exp(-\alpha_i k) \cos \left( k t_i + \frac{\beta_i \pi}{2} \right) dt_i
$$

$$
+ \sum_{r=2}^{m} (-1)^{r+1} \sum_{\mu(r) \in \mathbb{N}} \frac{1}{\pi} \int_{T_r} f_{\beta_r}^\psi \left( \bar{x} + \sum_{i \in \mu(r)} t_i \bar{e}_i \right)
$$

$$
\times \prod_{j \in \mu(r)} \sum_{k_j = n_j+1}^{\infty} \exp(-\alpha_{s_j} k_j) \cos \left( k_j t_j + \frac{\beta_{s_j} \pi}{2} \right) dt_j.
$$
Denote \( q_i = \exp(-\alpha_i) \), \( Q_i = \exp(-\alpha_i') \). Using [15, p. 123–124] we obtain

\[
\sum_{k=n_i+1}^{\infty} \exp(-\alpha_i k) \cos \left( kt_i + \frac{\beta_i \pi}{2} \right) = q_i^{n_i} \left[ \frac{q_i \cos t_i - q_i^2}{1 - 2q_i \cos t_i + q_i^2} \cdot \frac{q_i \sin t_i}{1 - 2q_i \cos t_i + q_i^2} \right].
\]

If

\[
h_{n_i}^\beta(t_i) = \frac{(q_i \cos t_i - q_i^2) \cos (n_i t_i + \frac{\beta_i \pi}{2}) - q_i \sin t_i \sin (n_i t_i + \frac{\beta_i \pi}{2})}{1 - 2q_i \cos t_i + q_i^2},
\]

\[
H_{n_i}^{\beta'}(t_i) = \frac{(Q_i \cos t_i - Q_i^2) \cos (n_i t_i + \frac{\beta_i \pi}{2}) - Q_i \sin t_i \sin (n_i t_i + \frac{\beta_i \pi}{2})}{1 - 2Q_i \cos t_i + Q_i^2}
\]

then

\[
\rho_\pi(f, \bar{x}) = \sum_{i=1}^{m} \frac{1}{\pi} \int_{-\pi}^{\pi} f_{\beta_i}^\nu_i(\bar{x} + t_i \bar{e}_i) q_i^{n_i} h_{n_i}^{\beta_i}(t_i) dt_i
\]

\[
+ \sum_{r=2}^{m} (-1)^{r+1} \sum_{\mu(\pi) \in \mathbb{Z}^r} \frac{1}{\pi^r} \int_{\pi}^{\pi} f_{\beta_i}^\nu_i(\bar{x} + \sum_{i \in \mu(\pi)} t_i \bar{e}_i) \prod_{j \in \mu(\pi)} Q_j^{\nu_j} H_j^{\beta_j}(t_j) dt_j.
\]

According to (10) we obtain

\[
\delta_{\pi, \nu}(f, \bar{x}) = \sum_{i=1}^{m} \frac{1}{\pi} \prod_{k_i=n_i-p_i} f_{\beta_i}^\nu_i(\bar{x} + t_i \bar{e}_i) h_{k_i}^{\beta_i}(t_i) dt_i
\]

\[
+ \sum_{r=2}^{m} (-1)^{r+1} \sum_{\mu(\pi) \in \mathbb{Z}^r} \frac{1}{\pi^r} \int_{\pi}^{\pi} f_{\beta_i}^\nu_i(\bar{x} + \sum_{i \in \mu(\pi)} t_i \bar{e}_i)
\]

\[
\times \prod_{j \in \mu(\pi)} \frac{1}{\pi} \sum_{n_j-1}^{n_j-1} Q_j^{\nu_j} H_j^{\beta_j}(t_j) dt_j.
\]

(11)

Let us use [11, p. 232–234]. Applying elementary transformations we obtain

\[
\sum_{k_i=n_i-p_i}^{n_i-1} q_i^{k_i} h_{k_i}^{\beta_i}(t) = \sum_{k_i=n_i-p_i}^{n_i-1} q_i^{k_i+1} \left[ (\cos(k_i+1)t - q_i \cos k_i t) \cos \frac{\beta_i \pi}{2} - (\sin(k_i+1)t - q_i \sin k_i t) \sin \frac{\beta_i \pi}{2} \right] (1 - 2q_i \cos t + q_i^2)^{-1}
\]

\[
\frac{df}{dt} = \sum_{i=1}^{\Sigma_1(t)} \cos \frac{\beta_i \pi}{2} - \sum_{i=2}^{\Sigma_2(t)} \sin \frac{\beta_i \pi}{2}.
\]

(12)
Let us investigate $\Sigma_{t,1}(t)$ and $\Sigma_{t,2}(t)$. We may write

\[
\Sigma_{t,1}(t) = \sum_{k=n-p}^{n-1} q^{k+1}(\cos(k+1)t - q \cos kt) = \frac{1}{2} \left[ \sum_{k=0}^{n} (qe^{it})^{k} - \sum_{k=0}^{n-p} (qe^{-it})^{k} \right] + \frac{1}{2} \left[ \sum_{k=0}^{n} (qe^{-it})^{k} - \sum_{k=0}^{n-p} (qe^{it})^{k} \right] - \frac{q^{2}}{2} \left[ \sum_{k=0}^{n-1} (qe^{it})^{k} - \sum_{k=0}^{n-p-1} (qe^{-it})^{k} \right].
\]

According to [15, p. 124] we denote

\[
\Gamma(t) = (1 - 2q \cos t + q^{2})^{-1}.
\]  

(13)

Now we have

\[
\Sigma_{t,1}(t) = (q^{n+2} \cos nt - q^{n+1} \cos(n+1)t - q^{n-p+2} \cos(n-p)t + q^{n-p+1} \cos(n-p+1)t - q^{n+1} \cos(n-1)t - q^{n} \cos nt - q^{n-p+1} \cos(n-p-1)t + q^{n-p} \cos(n-p)t) \Gamma(t)
\]

\[
= (2q^{n+2} \cos nt - 2q^{n-p+2} \cos(n-p)t - q^{n+1} \cos(n+1)t + q^{n-p+1} \cos(n-p+1)t - q^{n+1} \cos(n-1)t - q^{n} \cos nt - q^{n-p+1} \cos(n-p-1)t + q^{n-p} \cos(n-p)t) \Gamma(t)
\]

(14)

Doing elementary transformation of the term in brackets on the right part of equality (14) we have

\[
2q^{n+2} \cos nt - q^{n+1} \cos(n+1)t - q^{n+3} \cos(n-1)t
\]

\[
= q^{n+1} ((2q - \cos t - q^{2} \cos t) \cos nt + (\sin t - q^{2} \sin t) \sin nt),
\]

(15)

\[
2q^{n-p+2} \cos(n-p)t - q^{n-p+1} \cos(n-p+1)t - q^{n-p+3} \cos(n-p-1)t
\]

\[
= q^{n-p+1} ((2q - \cos t - q^{2} \cos t) \cos(n-p)t + (\sin t - q^{2} \sin t) \sin(n-p)t).
\]

(16)
Comparing (13)–(16) we obtain
\[
\Sigma_1(t) = \left[q^{n+1}((2q - \cos t - q^2 \cos t) \cos nt + (\sin t - q^2 \sin t) \sin nt)
- q^{n-p+1}((2q - \cos t - q^2 \cos t) \cos(n-p)t
+ (\sin t - q^2 \sin t) \sin(n-p)t)\right](1 - 2q \cos t + q^2)^{-1}. \tag{17}
\]

Analogously, we may find
\[
\Sigma_2(t) = \left[q^{n+1}((2q^2 \sin t - \sin t) \cos nt + (2q - \cos t - q^2 \cos t) \sin nt)
- q^{n-p+1}((2q^2 \sin t - \sin t) \cos(n-p)t
+ (2q - \cos t - q^2 \cos t) \sin(n-p)t)\right](1 - 2q \cos t + q^2)^{-1}. \tag{18}
\]

Respecting the last relation we may the equality (12) write in the following way
\[
\frac{1}{p_i} \sum_{k_i=n_i-p_i}^{n_i-1} q_i^{k_i} h_{k_i}^{(n_i)}(t_i) = \frac{1}{p_i} \left[q_i^{n_i-p_i+1} \left(q_i^2 \cos t_i - 2q_i + \cos t_i\right) \cos\left((n_i-p_i)t_i + \frac{\beta_i \pi}{2}\right)
+ (q_i^2 \sin t_i - \sin t_i) \sin\left((n_i-p_i)t_i + \frac{\beta_i \pi}{2}\right) \right]\right](1 - 2q_i \cos t_i + q_i^2)^{-2}
- \frac{1}{p_i} \left[q_i^{n_i-p_i+1} \left(q_i^2 \cos t_i - 2q_i + \cos t_i\right) \cos\left(n_i t_i + \frac{\beta_i \pi}{2}\right)
+ (q_i^2 \sin t_i - \sin t_i) \sin\left(n_i t_i + \frac{\beta_i \pi}{2}\right) \right]\right](1 - 2q_i \cos t_i + q_i^2)^{-2}. \tag{19}
\]

Analogously,
\[
\frac{1}{p_i} \sum_{k_i=n_i-p_i}^{n_i-1} Q_i^{k_i} H_{k_i}^{(n_i)}(t_i) =
\frac{1}{p_i} \left[Q_i^{n_i-p_i+1} \left(Q_i^2 \cos t_i - 2Q_i + \cos t_i\right) \cos\left((n_i-p_i)t_i + \frac{\beta_i \pi}{2}\right)
+ (Q_i^2 \sin t_i - \sin t_i) \sin\left((n_i-p_i)t_i + \frac{\beta_i \pi}{2}\right) \right]\right](1 - 2Q_i \cos t_i + Q_i^2)^{-2}
- \frac{1}{p_i} \left[Q_i^{n_i-p_i+1} \left(Q_i^2 \cos t_i - 2Q_i + \cos t_i\right) \cos\left(n_i t_i + \frac{\beta_i \pi}{2}\right)
+ (Q_i^2 \sin t_i - \sin t_i) \sin\left(n_i t_i + \frac{\beta_i \pi}{2}\right) \right]\right](1 - 2Q_i \cos t_i + Q_i^2)^{-2}. \tag{20}
\]

Considering the condition
\[
\text{ess sup}_{\bar{x} \in T^m} |f^\mu (\bar{x})| \leq 1, \quad \mu \subset \mathbb{N}, \quad f \in C_{e, \infty}^m
\]
and equalities (11), (19), (20) we have the coreness the theorem. \(\square\)
3 Conclusion

Using the relation (9) we can obtain an asymptotic equality for upper bounds of the deviations of the de la Vallee Poussin right-angled sums taken over classes of periodic functions of several variables with a high smoothness.

References