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Dually Residuated ℓ-monoids Having No Non-trivial Convex Subalgebras*

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Abstract

In this note we describe the structure of dually residuated ℓ-monoids ($DR\ell$-monoids) that have no non-trivial convex subalgebras.

Key words: $DR\ell$-monoid; $GPMV$-algebra; Archimedean property.

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A *dually residuated ℓ-monoid*, a *$DR\ell$-monoid* for short, is an algebra

$$(A, \oplus, 0, \vee, \wedge, \oslash, \otimes)$$

of type $\langle 2, 0, 2, 2, 2, 2 \rangle$ such that

(a) $(A, \oplus, 0, \vee, \wedge)$ is a lattice-ordered monoid, i.e., $(A, \oplus, 0)$ is a monoid, $(A, \vee, \wedge)$ is a lattice and $\oplus$ distributes over both $\vee$ and $\wedge$,

(b) for any $a, b \in A$, $a \oslash b$ is the least element $x \in A$ with $x \oplus b \geq a$, and $a \otimes b$ is the least element $y \in A$ with $b \oplus y \geq a$, and

(c) A satisfies the identities

$$((x \oslash y) \vee 0) \oplus y \leq x \vee y, \quad y \oplus ((x \otimes y) \vee 0) \leq x \vee y,$$
$$x \oslash x \geq 0, \quad x \otimes x \geq 0.$$

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