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A Decomposition of Homomorphic Images of Nearlattices*

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Abstract

By a nearlattice is meant a join-semilattice where every principal filter is a lattice with respect to the induced order. The aim of our paper is to show for which nearlattice $\mathcal{S}$ and its element c the mapping $\varphi_c(x) = \langle x \vee c, x \wedge_p c\rangle$ is a (surjective, injective) homomorphism of $\mathcal{S}$ into $[c) \times (c]$.

Key words: Nearlattice; semilattice; distributive element; pseudocomplement; dual pseudocomplement.

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It is well-known (see e.g. [4]) that if L is a bounded distributive lattice and $c \in L$ has a complement in L then L is isomorphic to the direct product $[c) \times (c]$. On the other hand, if c is not complemented then the mapping $\varphi_c(x) = \langle x \vee c, x \wedge c\rangle$ is still an injective homomorphism of L into the mentioned direct product and one can discuss whether the homomorphic image $\varphi_c(L)$ is a subdirect product of $[c) \times (c]$.

In what follows we generalize this setting for the so-called nearlattices (see [1–3, 5–8]) and we investigate which of these results remain true. It turns out that our task is reasonable only for a class of so-called nested nearlattices.

Definition 1 By a *nearlattice* we mean a semilattice $\mathcal{S} = (S; \vee)$ where for each $a \in S$ the principal filter $[a) = \{x \in S; a \le x\}$ is a lattice with respect to the induced order $\le$ of $\mathcal{S}$.

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