A Decomposition of Homomorphic Images of Nearlattices

IVAN CHAJDA\textsuperscript{1}, MIROSLAV KOLAŘÍK\textsuperscript{2}

Department of Algebra and Geometry, Faculty of Science, Palacký University,
Tomkova 40, 779 00 Olomouc, Czech Republic
e-mail: \textsuperscript{1}chajda@inf.upol.cz
\textsuperscript{2}kolarik@inf.upol.cz

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Abstract

By a nearlattice is meant a join-semilattice where every principal filter is a lattice with respect to the induced order. The aim of our paper is to show for which nearlattice $S$ and its element $c$ the mapping $\phi_c(x) = \langle x \lor c, x \land c \rangle$ is a (surjective, injective) homomorphism of $S$ into $[c] \times (c)$.

Key words: Nearlattice; semilattice; distributive element; pseudo-complement; dual pseudocomplement.

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It is well-known (see e.g. [4]) that if $L$ is a bounded distributive lattice and $c \in L$ has a complement in $L$ then $L$ is isomorphic to the direct product $[c] \times (c)$. On the other hand, if $c$ is not complemented then the mapping $\varphi_c(x) = \langle x \lor c, x \land c \rangle$ is still an injective homomorphism of $L$ into the mentioned direct product and one can discuss whether the homomorphic image $\varphi_c(L)$ is a subdirect product of $[c] \times (c)$.

In what follows we generalize this setting for the so-called nearlattices (see [1–3, 5–8]) and we investigate which of these results remain true. It turns out that our task is reasonable only for a class of so-called nested nearlattices.

Definition 1 By a nearlattice we mean a semilattice $S = (S; \lor)$ where for each $a \in S$ the principal filter $[a] = \{ x \in S; a \leq x \}$ is a lattice with respect to the induced order $\leq$ of $S$.

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