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Problems Concerning Subclasses of Analytic Functions *

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Abstract

In the present paper, we establish some interesting results concerning the quasi-Hadamard product for certain subclasses of analytic functions.

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1 Introduction and preliminaries

Throughout this paper, let the of functions of the form

$$f(z) = a_1 z + \sum_{n=2}^{\infty} a_n z^n, \quad a_1 > 0,\ a_n \geq 0, \tag{1}$$

$$f_i(z) = a_{1,i} z + \sum_{n=2}^{\infty} a_{n,i} z^n, \quad a_{1,i} > 0,\ a_{n,i} \geq 0, \tag{2}$$

$$g(z) = b_1 z + \sum_{n=2}^{\infty} b_n z^n, \quad b_1 > 0,\ b_n \geq 0, \tag{3}$$

$$g_j(z) = b_{1,j} z + \sum_{n=2}^{\infty} b_{n,j} z^n, \quad b_{1,i} > 0,\ b_{n,j} \geq 0, \tag{4}$$

be regular and univalent in the unit disc $\mathbb{U} = \{z\colon |z| < 1\}$.

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For a function $f(z)$ defined by (1) (with $a_1 = 1$) we define

$$\begin{aligned} D^0 f(z) &= f(z), \\ D^1_\lambda(\alpha,\beta,\mu)f(z) &= (\tfrac{\alpha-\mu+\beta-\lambda}{\alpha+\beta})f(z) + (\tfrac{\mu+\lambda}{\alpha+\beta})zf'(z), \\ D^2_\lambda(\alpha,\beta,\mu)f(z) &= D(D^1_\lambda(\alpha,\beta,\mu)f(z)), \\ &\vdots \\ D^k_\lambda(\alpha,\beta,\mu)f(z) &= D(D^{k-1}_\lambda(\alpha,\beta,\mu)f(z)). \end{aligned} \tag{5}$$

If f is given by (1) then from (5) we have

$$D^k_\lambda(\alpha,\beta,\mu)f(z) = z + \sum_{n=2}^{\infty}\Big(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\Big)^k a_n z^n, \tag{6}$$

where $f \in A$, $\alpha,\beta,\mu,\lambda \geq 0$, $\alpha+\beta \neq 0, n \in N_o$. The operator $D^k_\lambda(\alpha,\beta,\mu)f(z)$ was introduced by authors, Darus and Faisal in [7].

By specializing the parameters of $D^n_\lambda(\alpha,\beta,\mu)f(z)$ we get the following differential operators. If we substitute

• $\beta = 0$, we get $D^k_\lambda(\alpha,0,\mu)f(z) = z + \sum_{n=2}^{\infty}(\frac{\alpha+(\mu+\lambda)(n-1)}{\alpha})^k a_n z^n$ of differential operator given by Darus and Faisal [8].

• $\beta = 1$, $\mu = 0$, we get $D^k_\lambda(\alpha,1,0)f(z) = z + \sum_{n=2}^{\infty}(\frac{\alpha+\lambda(n-1)+1}{\alpha+1})^k a_n z^n$ of differential operator given by Aouf et. al [3].

• $\alpha = 1$, $\beta = o$, and $\mu = 0$, we get $D^k_\lambda(1,0,0)f(z) = z + \sum_{n=2}^{\infty}(1+\lambda(n-1))^k a_n z^n$ of differential operator given by Al-Oboudi [1].

• $\alpha = 1$, $\beta = o$, $\mu = 0$ and $\lambda = 1$, we get $D^k_1(1,0,0)f(z) = z + \sum_{n=2}^{\infty}(n)^k a_n z^n$ of differential operator given by Sălăgean's [12].

• $\alpha = 1$, $\beta = 1$, $\lambda = 1$ and $\mu = 0$, we get $D^k_1(1,1,0)f(z) = z + \sum_{n=2}^{\infty}(\frac{n+1}{2})^k a_n z^n$ of differential operator given by Uralegaddi and Somanatha [16].

• $\beta = 1$, $\lambda = 1$ and $\mu = 0$, we get $D^k_1(\alpha,1,0)f(z) = z + \sum_{n=2}^{\infty}(\frac{n+\alpha}{\alpha+1})^k a_n z^n$ of differential operator given by Cho and Srivastava [5, 6].

With the help of the differential operator $D^k_\lambda(\alpha,\beta,\mu)f(z)$, we define the classes $\Psi(k,\rho,\delta,\eta)$, and $R_p(n,\rho,\delta,\eta)$ as follows:

For, $0 \leq \rho < 1$, $0 \leq \delta < 1$ and $\eta \geq 0$, we let $\Psi(k,\rho,\delta,\eta)$ denote the class of functions $f(z)$ defined by (1) and satisfying the analytic criterion

$$\begin{aligned} &\Re\Big\{\frac{z(D^k_\lambda(\alpha,\beta,\mu)f(z))'}{(1-\rho)(D^k_\lambda(\alpha,\beta,\mu)f(z)) + \rho z(D^k_\lambda(\alpha,\beta,\mu)f(z))'} - \delta\Big\} \\ &> \eta\Big|\frac{z(D^k_\lambda(\alpha,\beta,\mu)f(z))'}{(1-\rho)(D^k_\lambda(\alpha,\beta,\mu)f(z)) + \rho z(D^k_\lambda(\alpha,\beta,\mu)f(z))'} - 1\Big|, \ z \in \mathbb{U}. \end{aligned} \tag{7}$$

Suitably specializing the parameters of the class $\Psi(k,\rho,\delta,\eta)$ generalize the classes defined by some well known authors see [13]–[15].

Using similar arguments as given in [11], we can easily get the following results for functions in the classes $\Psi(n,\rho,\delta,\eta)$ and $\psi(k+1,\rho,\delta,\eta)$ respectively.

A function $f(z)\in\Psi(k,\rho,\delta,\eta)$ $(0\le\rho<1, 0\le\delta<1, \eta\ge 0)$ if and only if

$$\sum_{n=2}^{\infty}\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{k}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]|a_n| \le (1-\delta), \tag{8}$$

Now we introduce the following class of analytic functions which plays an important role in our discussion as follows.

A function $f(z)$ which is analytic in U belongs to the class $R_p(n,\rho,\delta,\eta)$ if and only if

$$\sum_{n=2}^{\infty}\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{p}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]|a_n| \le (1-\delta), \tag{9}$$

where $0\le\rho<1$, $0\le\delta<1$, $\eta\ge 0$ and p is any fixed nonnegative real number.

For $p=k$, it is identical to $\Psi(k,\rho,\delta,\eta)$. Also for $p=\beta=\mu=0$, it is identical to $S_p(\rho,\delta,\eta)$ defined in [11]. Further, for any positive integer $p>h>h-1>\cdots>k+1>k$, we have the inclusion relation

$$R_p(n,\rho,\delta,\eta)\subseteq R_h(n,\rho,\delta,\eta)\subseteq\cdots\subseteq\psi(k+1,\rho,\delta,\eta)\subseteq\Psi(k,\rho,\delta,\eta).$$

The class $R_p(n,\rho,\delta,\eta)$ is nonempty for any nonnegative real number p as the functions of the form

$$f(z)=a_1z+\sum_{n=2}^{\infty}\frac{(\alpha+\beta)^p(1-\delta)}{\left(\alpha+(\mu+\lambda)(n-1)+\beta\right)^{p}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]}\lambda_n z^n, \tag{10}$$

where $a_1>0$, $\lambda_n\ge 0$ and $\Sigma_2^{\infty}\lambda_n\le 1$; satisfy the inequality (10).

Let us define the quasi-Hadamard product of the functions $f(z)$ and $g(z)$ by

$$f*g(z)=a_1b_1z+\sum_{n=2}^{\infty}a_nb_nz^n,\quad a_1>0,\ b_1>0,\ a_n\ge 0. \tag{11}$$

In this work we establish certain results concerning the quasi-Hadamard product of functions for the classes $\Psi(k,\rho,\delta,\eta)$ and $R_p(n,\rho,\delta,\eta)$ analogous to the results due to V. Kumar [9] and [10] as well.

2 The main results

Theorem 1 *Let the functions* $f_i(z)$ *defined by (2) be in the class* $\psi(k+1,\rho,\delta,\eta)$ *for every* $i = 1, 2, \cdots, r$ *and let the functions* $g_j(z)$ *defined by (4) be in the class* $\Psi(n,\rho,\delta,\eta)$ *for every* $j = 1, 2, \cdots, q$. *Then the quasi-Hadamard product* $f_1 * f_2 * \cdots * f_r * g_1 * g_2 * \cdots * g_q(z)$ *belongs to the class* $R_{r(k+2)+q(k+1)-1}(n,\rho,\delta,\eta)$.

Proof We denote the quasi-Hadamard product $f_1 * f_2 * \cdots * f_r * g_1 * g_2 * \cdots * g_q(z)$ by the function $G(z)$ for the sake of convenience. Clearly,

$$G(z) = \left[\prod_{i=1}^{r} |a_{1,i}|\right]\left[\prod_{j=1}^{q} |b_{1,j}|\right] z + \sum_{n=2}^{\infty} \left[\prod_{i=1}^{r} |a_{n,i}|\right]\left[\prod_{j=1}^{q} |b_{n,j}|\right] z^n.$$

Since $f_i(z) \in \psi(k+1,\rho,\delta,\eta)$ implies

$$\sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{k+1} \big[n(1+\eta) - (\delta+\eta)(1+n\rho-\rho)\big] |a_{n,i}| \leq (1-\delta)|a_{1,i}|, \tag{12}$$

for every $i = 1, 2, \cdots, r$. Implies

$$|a_{n,i}| \leq \frac{(\alpha+\beta)^{k+1}(1-\delta)}{\Big(\alpha + (\mu+\lambda)(n-1)+\beta\Big)^{k+1} \big[n(1+\eta) - (\delta+\eta)(1+n\rho-\rho)\big]} |a_{1,i}|,$$

for every $i = 1, 2, \cdots, r$. Implies

$$|a_{n,i}| \leq \left(\frac{\alpha + (\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{-k-2} |a_{1,i}|, \tag{13}$$

for every $i = 1, 2, \cdots, r$. Similarly $g_j(z) \in \Psi(k,\rho,\delta,\eta)$, implies

$$\sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{k} \big[n(1+\eta) - (\delta+\eta)(1+n\rho-\rho)\big] |b_{n,j}| \leq (1-\delta)|b_{1,j}|, \tag{14}$$

for every $i = 1, 2, \cdots q$. Implies

$$|b_{n,j}| \leq \left(\frac{\alpha + (\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{-k-1} |b_{1,j}|, \tag{15}$$

for every $j = 1, 2, \cdots, q$. To prove the theorem, we need to show that

$$\sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{t} \big[n(1+\eta) - (\delta+\eta)(1+n\rho-\rho)\big] \times \left[\prod_{i=1}^{r} |a_{n,i}|\right]\left[\prod_{j=1}^{q} |b_{n,j}|\right] \leq (1-\delta)\left[\prod_{i=1}^{r} |a_{1,i}|\right]\left[\prod_{j=1}^{q} |b_{1,j}|\right],$$

where $t = r(k+2) + q(k+1) - 1$.

Using (13), (14) and (15) for $i = 1, 2, \cdots, r$, $j = q$ and $j = 1, 2, \cdots, q-1$ respectively. we have (consider $t = r(k+2) + q(k+1) - 1$)

$$\sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu+\lambda)(n-1) + \beta}{\alpha+\beta} \right)^t \big[n(1+\eta) - (\delta+\eta)(1+n\rho-\rho) \big]$$

$$\times \left[\prod_{i=1}^{r} |a_{n,i}| \right] \left[\prod_{j=1}^{q} |b_{n,j}| \right]$$

$$\leq \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu+\lambda)(n-1) + \beta}{\alpha+\beta} \right)^{r(k+2)+q(k+1)-1} \big[n(1+\eta) - (\delta+\eta)(1+n\rho-\rho) \big]$$

$$\times \left[\left(\frac{\alpha + (\mu+\lambda)(n-1) + \beta}{\alpha+\beta} \right)^{-r(k+2)} \left(\frac{\alpha + (\mu+\lambda)(n-1) + \beta}{\alpha+\beta} \right)^{-(q-1)(k+1)} \right.$$

$$\left. \times |b_{n,q}| \prod_{i=1}^{r} |a_{1,i}| \right] \left[\prod_{j=1}^{q-1} |b_{1,j}| \right] = \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu+\lambda)(n-1) + \beta}{\alpha+\beta} \right)^k$$

$$\times \big[n(1+\eta) - (\delta+\eta)(1+n\rho-\rho) \big] |b_{n,q}| \left[\prod_{i=1}^{r} |a_{1,i}| \right] \left[\prod_{j=1}^{q-1} |b_{1,j}| \right]$$

$$\leq (1-\delta) \left[\prod_{i=1}^{r} |a_{1,i}| \right] \left[\prod_{j=1}^{q} |b_{1,j}| \right].$$

Implies

$$\sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu+\lambda)(n-1) + \beta}{\alpha+\beta} \right)^t \big[n(1+\eta) - (\delta+\eta)(1+n\rho-\rho) \big]$$

$$\times \left[\prod_{i=1}^{r} |a_{n,i}| \right] \left[\prod_{j=1}^{q} |b_{n,j}| \right]$$

$$\leq (1-\delta) \left[\prod_{i=1}^{r} |a_{1,i}| \right] \left[\prod_{j=1}^{q} |b_{1,j}| \right], \quad t = r(k+2) + q(k+1) - 1.$$

Using (9) we conclude that

$$f_1 * f_2 * \cdots * f_r * g_1 * g_2 * \cdots * g_q(z) \in R_{r(k+2)+q(k+1)-1}(n, \rho, \delta, \eta). \qquad \square$$

Theorem 2 *Let the functions $f_i(z)$ defined by (2) be in the class $\psi(k+1, \rho, \delta, \eta)$ for every $i = 1, 2, \cdots, r$. Then the Hadamard product $f_1(z) * f_2(z) * \cdots * f_r(z)$ belongs to the class $R_{r(k+2)-1}(n, \rho, \delta, \eta)$.*

Proof To prove the theorem, we need to show that

$$\sum_{n=2}^{\infty}\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{r(k+2)-1}\big[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)\big]\prod_{i=1}^{r}|a_{n,i}| \leq (1-\delta)\prod_{i=1}^{r}|a_{1,i}|.$$

Since $f_i(z)\in\psi(k+1,\rho,\delta,\eta)$, implies

$$\sum_{n=2}^{\infty}\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{k+1}\big[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)\big]|a_{n,i}| \leq (1-\delta)|a_{1,i}|, \tag{16}$$

for every $i=1,2,\cdots,r$. Implies

$$|a_{n,i}|\leq\frac{(\alpha+\beta)^{k+1}(1-\delta)}{\big(\alpha+(\mu+\lambda)(n-1)+\beta\big)^{k+1}\big[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)\big]}|a_{1,i}|,$$

for every $i=1,2,\cdots,r$. Implies

$$|a_{n,i}|\leq\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{-k-2}|a_{1,i}|, \tag{17}$$

for every $i=1,2,\cdots,r$.

Simultaneously applying (16) as well as (17) for $i=r$ and $i=1,2,\cdots,r-1$ respectively, we get

$$\sum_{n=2}^{\infty}\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{r(k+2)-1}\big[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)\big]\prod_{i=1}^{r}|a_{n,i}|$$

$$\leq\sum_{n=2}^{\infty}\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{r(k+2)-1}\big[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)\big]|a_{n,r}|$$

$$\times\left[\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{-(r-1)(k+2)}\prod_{i=1}^{r-1}|a_{1,i}|\right]$$

$$=\sum_{n=2}^{\infty}\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{k+1}\big[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)\big]|a_{n,r}|\left[\prod_{i=1}^{r-1}|a_{1,i}|\right]$$

$$=(1-\delta)\prod_{i=1}^{r}|a_{1,i}|.$$

Hence $f_1(z)*f_2(z)*\cdots*f_r(z)\in R_{r(k+2)-1}(k,\rho,\delta,\eta)$. □

Theorem 3 *Let the functions $f_i(z)$ defined by (2) be in the class $\Psi(k,\rho,\delta,\eta)$ for every $i=1,2,\cdots,r$. Then the Hadamard product $f_1(z)*f_2(z)*\cdots*f_r(z)$ belongs to the class $R_{r(k+1)-1}(n,\rho,\delta,\eta)$.*

Proof Since $f_i(z) \in \Psi(k,\rho,\delta,\eta)$, implies

$$\sum_{n=2}^{\infty}\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{k}\big[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)\big]|a_{n,i}| \le (1-\delta)|a_{1,i}|, \tag{18}$$

for every $i=1,2,\cdots,r$. Implies

$$\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{k}\big[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)\big]|a_{n,i}| \le (1-\delta)|a_{1,i}|,$$

or

$$|a_{n,i}| \le \frac{(1-\delta)}{\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{k}\big[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)\big]}|a_{1,i}|,$$

for every $i=1,2,\cdots,r$. Implies

$$|a_{n,i}| \le \left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{-k-1}|a_{1,i}|, \tag{19}$$

for every $i=1,2,\cdots,r$.

To prove that $f_1(z)*f_2(z)*\cdots*f_r(z) \in R_{r(k+1)-1}(n,\rho,\delta,\eta)$, it is enough to show that

$$\sum_{n=2}^{\infty}\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{r(k+1)-1}\big[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)\big]\prod_{i=1}^{r}|a_{n,i}| \le (1-\delta)\prod_{i=1}^{r}|a_{1,i}|. \tag{20}$$

Using (18) and (19) for $i=r$ and $i=1,2,\cdots,r-1$ respectively, then we have

$$\begin{aligned}&\sum_{n=2}^{\infty}\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{r(k+1)-1}\big[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)\big]\prod_{i=1}^{r}|a_{n,i}|\\ &\le \sum_{n=2}^{\infty}\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{r(k+1)-1}\big[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)\big]|a_{n,r}|\\ &\quad\times\Bigg[\left(\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta}\right)^{-(r-1)(k+1)}\prod_{i=1}^{r-1}|a_{1,i}| = (1-\delta)\prod_{i=1}^{r}|a_{1,i}|.\end{aligned}$$

Hence $f_1(z)*f_2(z)*\cdots*f_r(z) \in R_{r(k+1)-1}(n,\rho,\delta,\eta)$. □

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