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Class Preserving Mappings of Equivalence Systems

Ivan CHAJDA

*Department of Algebra and Geometry, Faculty of Science,
Palacký University, Tomkova 40, 779 00 Olomouc, Czech Republic
e-mail: chajda@inf.upol.cz*

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Abstract

By an equivalence system is meant a couple $\mathcal{A} = (A, \theta)$ where A is a non-void set and θ is an equivalence on A. A mapping h of an equivalence system $\mathcal{A}$ into $\mathcal{B}$ is called a class preserving mapping if $h([a]_\theta) = [h(a)]_{\theta'}$ for each $a \in A$. We will characterize class preserving mappings by means of permutability of θ with the equivalence Φ_h induced by h.

Key words: Equivalence relation, equivalence system, relational system, homomorphism, strong homomorphism, permuting equivalences.

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For the basic concepts, the reader is referred to [1],[2],[3]. Let R and S be binary relations on a non-void set A. As usually, their *relational product* will be denoted by $R \circ S$, i.e. $R \circ S = \{\langle a, b\rangle \in A^2; \exists c \in A \text{ with } \langle a, c\rangle\rangle \in R$ and $\langle c, b\rangle \in S\}$. We will say that R, S *permute* (or they are *permutable*) if $R \circ S = S \circ R$.

Lemma 1 *Let R, S be symmetric relations on A. Then $R \circ S \subseteq S \circ R$ is equivalent to $R \circ S = S \circ R$.*

Proof If $R \circ S \subseteq S \circ R$ then, due to symmetry,

$$S \circ R = S^{-1} \circ R^{-1} = (R \circ S)^{-1} \subseteq (S \circ R)^{-1} = R^{-1} \circ S^{-1} = R \circ S$$

thus S, R permute. The converse is trivial. □