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Equipping Distributions for Linear Distribution*

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Abstract

In this paper there are discussed the three-component distributions of affine space A_{n+1}. Functions $\{\mathcal{M}^\sigma\}$, which are introduced in the neighborhood of the second order, determine the normal of the first kind of $\mathcal{H}$-distribution in every center of $\mathcal{H}$-distribution.

There are discussed too normals $\{\mathcal{Z}^\sigma\}$ and quasi-tensor of the second order $\{\mathcal{S}^\sigma\}$. In the same way bunches of the projective normals of the first kind of the $\mathcal{M}$-distributions were determined in the differential neighborhood of the second and third order.

Key words: Equipping distributions; linear distribution; affine space.

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1 Introduction

The given paper applies to differential geometry of a multi-dimensional affine space A_{n+1}. The three-component distributions of an affine space are discussed.

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Functions $\{\mathcal{M}^\sigma\}$ are introduced in the neighborhood of the second order. They determine the normal of the first kind of a $\mathcal{H}$-distribution in every center of a $\mathcal{H}$-distribution. The normal $\{\mathcal{M}^\sigma\}$ is a generalization of Miheylesku normal of the first kind for a hyperplane distribution of an affine space. The field of the normals $\{\mathcal{Z}^\sigma\}$ was constructed by an inner invariant method in the third differential neighborhood of the forming element of the $\mathcal{H}$-distribution. The object $\{Z^\sigma\}$ determines the projective normal – analog of Fubini normal for the $\mathcal{H}$-distribution in every center of the forming element of the $\mathcal{H}$-distribution. The quasi-tensor of the second order $\{S^\sigma\}$ determines the projective normal of the first kind of the $\mathcal{H}$-distribution. Projective normals of the first kind $\{\mathcal{M}^\sigma\}$, $\{\mathcal{Z}^\sigma\}$, $\{S^\sigma\}$ determine bunches of the projective normals of the first kind of the $\mathcal{H}$-distribution in the differential neighborhood of the second and third orders. In the same way bunches of the projective normals of the first kind of the M-distribution were determined in the differential neighborhood of the second and third orders. We use results, which we have got in [2, 3].

2 Definition of the three-component distribution

Let us consider an $(n+1)$-dimensional affine space A_{n+1}, which is taken to a movable frame $R = \{A, \bar{e}_\mathcal{I}\}$. Differential equations of an infinitesimal transference of the frame R look as follows: $dA = \omega^\mathcal{I}\bar{e}_\mathcal{I}$, $\quad d\bar{e}_\mathcal{I} = \omega^\mathcal{K}_\mathcal{I}\bar{e}_\mathcal{K}$, where $\omega^\mathcal{K}_\mathcal{I}$, $\omega^\mathcal{I}$ are invariant forms of an affine group, which satisfy equations of the structure:

$$d\omega^\mathcal{I} = \omega^\mathcal{K} \wedge \omega^\mathcal{I}_\mathcal{K}, \qquad d\omega^\mathcal{K}_\mathcal{I} = \omega^\mathcal{J}_\mathcal{I} \wedge \omega^\mathcal{K}_\mathcal{J}.$$

Structural forms of a current point $X = A + x^\mathcal{I}\bar{e}_\mathcal{I}$ of a space A_{n+1} look as follows:

$$\triangle X^\mathcal{I} \equiv dx^\mathcal{I} + x^\mathcal{K}\omega^\mathcal{I}_\mathcal{K} + \omega^\mathcal{I}.$$

The combination of the current point X and point of the frame A leads to the following equation:

$$\triangle X^\mathcal{I} = \omega^\mathcal{I}.$$

An immobility condition of the point A is written down as follows: $\omega^\mathcal{I} = 0$.

Let the frame chosen by this way be called the frame $\tilde{R}$. Let Π_r is an r-dimensional plane in A_{n+1} be given by the following way: $\Pi_r = [A, \bar{L}_p]$, where $\bar{L}_p = \bar{e}_p + \Lambda^{\hat{u}}_p\bar{e}_{\hat{u}}$. Let m-dimensional plane Π_m be set by the following way: $\Pi_m = [A, \bar{M}_a]$, where $\bar{M}_a = \bar{e}_a + M^{\hat{\alpha}}_a\bar{e}_{\hat{\alpha}}$. A hyperplane Π_n is a set $\Pi_n = [A, \bar{T}_\sigma]$, where $\bar{T}_\sigma = \bar{e}_\sigma + H^{n+1}_\sigma\bar{e}_{n+1}$.

Definition 1 The $(n+1)$-dimensional manifolds in spaces of notion $\{\triangle\Lambda^{\hat{u}}_p, \omega^\mathcal{I}\}$, $\{\triangle M^{\hat{\alpha}}_a, \omega^\mathcal{I}\}$, $\{\triangle H^{n+1}_\sigma, \omega^\mathcal{I}\}$ which are determined by differential equations

$$\triangle\Lambda^{\hat{u}}_p = \Lambda^{\hat{u}}_{p\mathcal{K}}\omega^\mathcal{K}, \quad \triangle M^{\hat{\alpha}}_a = M^{\hat{\alpha}}_{a\mathcal{K}}\omega^\mathcal{K}, \quad \triangle H^{n+1}_\sigma = H^{n+1}_{\sigma\mathcal{K}}\omega^\mathcal{K}, \tag{1}$$

are called *distributions of the first kind accordingly of: r-dimensional linear elements (Λ-distribution), m-dimensional linear elements (M-distribution)* and *hyperplanes (H-distribution).*

Equations of the system (1) to each point A (center of distribution) are the set according to planes Π_r, Π_m, Π_n.

Let consider that manifolds (1) are distributions of tangent elements: center A belongs to planes Π_r, Π_m, Π_n. We demand, that in some area of the space A_{n+1} for any center A the following condition take place: $A \in \Pi_r \subset \Pi_m \subset \Pi_n$.

Definition 2 The three of distributions of the affine space A_{n+1}, consisting of basic distribution of the first kind r-dimensional linear elements $\Pi_r \equiv \Lambda$ (Λ-*distribution*), equipping distribution of the first kind of m-dimensional linear elements $\Pi_m \equiv M$ (M-*distribution*) and equipping distribution of the first kind of hyperplane elements $\Pi_r \equiv H$ $(r < m < n)$ (H-*distribution*) with relation of an incidence of their corresponding elements in a common center A of the following view: $A \in \Lambda \subset M \subset H$ are called $\mathcal{H}$-*distribution*.

Let us make the following canonization of the frame $\tilde{R}$: we will place vectors $\bar{e}_p$ in the plane Π_r, vectors $\bar{e}_i$ – in plane Π_m, and vectors $\bar{e}_\sigma$ – in plane Π_n. Such frame will be called the frame of the null order R^0. This definition leads to the following equations:

$$\Lambda_p^{\hat{u}} = 0, \quad M_a^{\hat{\alpha}} = 0, \quad H_\sigma^{n+1} = 0.$$

In the frame R^0 the $\mathcal{H}$-distribution is defined by the differential equations:

$$\omega_p^{\hat{u}} = \Lambda_{p\mathcal{K}}^{\hat{u}} \omega^{\mathcal{K}}, \quad \omega_i^{\hat{\alpha}} = M_{i\mathcal{K}}^{\hat{\alpha}} \omega^{\mathcal{K}}, \quad \omega_\alpha^{n+1} = H_{\alpha\mathcal{K}}^{n+1} \omega^{\mathcal{K}}.$$

According to N. Ostianu lemma it is possible partial the zero-order frame R^0 canonization, where $M_{iq}^{n+1} = 0$, $H_{\alpha q}^{n+1} = 0$. We will call it frame of the first order R^1.

In the chosen frame R^1 the manifold $\mathcal{H}$ is determined by the following system of differential equations:

$$\omega_p^{\hat{u}} = \Lambda_{p\mathcal{K}}^{\hat{u}} \omega^{\mathcal{K}}, \quad \omega_i^{n+1} = M_{i\hat{u}}^{n+1} \omega^{\hat{u}},$$

$$\omega_i^{\alpha} = M_{i\mathcal{K}}^{\alpha} \omega^{\mathcal{K}}, \quad \omega_\alpha^{n+1} = H_{\alpha\hat{u}}^{n+1} \omega^{\hat{u}}, \quad \omega_u^p = A_{u\mathcal{K}}^p \omega^{\mathcal{K}}.$$

3 Tensor of inholonomicity of $\mathcal{H}$-distribution

It's easy to show, that geometry of three-component distributions can be used for studying geometry of regular and degenerate hyperzones, zones, hyperzone distributions, surfaces of full and not full range, tangent equipped surfaces in affine spaces. For example, we will suppose, that the $\mathcal{H}$-distribution is holonomic, that is the basic distribution Λ is holonomic. System of differential equations $\omega_0^{\hat{u}} = \Lambda_p^{\hat{u}} \omega_0^p$, which is associated with basic Λ-distribution, is quite integrable if and only if, when the tensor of the first order

$$r_{pq}^{\hat{u}} = \frac{1}{2}(\Lambda_{pq}^{\hat{u}} - \Lambda_{qp}^{\hat{u}})$$

turns into zero.

Tensor $\{r^{\hat{u}}_{pq}\}$ will be called tensor of the inholonomicity of the $\mathcal{H}$-distribution. The basic Λ-distribution determines $(n-r+1)$-parametric assemblage of r-dimensional surfaces V_r (planes Λ are rounded by r-dimensional surfaces of $(n-r+1)$-parametric assemblage).

In the time of displacement of center A along a fixed surface V_r, differential equations, which determine the $\mathcal{H}$-distribution relatively the frame $\tilde{R}$

$$\omega^{\hat{u}}_0 = \Lambda^{\hat{u}}_q \omega^q, \qquad \triangle\Lambda^{\hat{u}}_p = (\Lambda^{\hat{u}}_{pq} + \Lambda^{\hat{u}}_{p\hat{v}}\Lambda^{\hat{v}}_q)\omega^q,$$
$$\triangle M^{\hat{\alpha}}_i = (M^{\hat{\alpha}}_{iq} + M^{\hat{\alpha}}_{i\hat{v}}\Lambda^{\hat{v}}_q)\omega^q, \qquad \triangle H^{n+1}_\alpha = (H^{n+1}_{\alpha q} + H^{n+1}_{\alpha\hat{v}}\Lambda^{\hat{v}}_q)\omega^q$$

are differential equations of r-dimensional zone $V_{r(m)}$ of the order m [7, 8] equipped by a field of hyperplanes H. A geometrical object $\{H^{n+1}_r\}$ (object H) is the fundamental equipping object of a zone $V_{r(m)}$.

Following G. F. Laptev [5], the zone $V_{r(m)}$, on which the field of the fundamental equipping object H is set, we will call an equipped zone $V_{r(m)}$ and we will disignate as $V_{r(m)}(H)$.

Let note, that relatively of the frame R^0, which is adapted the fields of the planes Λ, M, H, differential equations of the manifold $V_{r(m)}(H)$ have more simple form:

$$\omega^{\hat{u}}_0 = 0, \quad \omega^{\hat{u}}_p = \Lambda^{\hat{u}}_{pq}\omega^q, \quad \Lambda^{\hat{u}}_{pq} = \Lambda^{\hat{u}}_{qp}, \tag{2}$$

$$\omega^{\hat{\alpha}}_i = M^{\hat{\alpha}}_{iq}\omega^q, \tag{3}$$

$$\omega^{n+1}_\alpha = H^{n+1}_{\alpha q}\omega^q, \tag{4}$$

where equations (2), (3) are analogous to equations of the zone $V_{r(m)}$, which are discussed in the work of M.M. Pohila [7]. Equations (4) characterize the equipment of the zone $V_{r(m)}$ by the field of hyperzones H.

Thus, a transformation of a tensor $\{r^{\hat{u}}_{pq}\}$ to zero is the condition, where the space A_{n+1} desintegrates to $(n-r+1)$-parametric assemblage of equipped zones $V_{r(m)}(H)$. So plane $\Lambda(A)$ in its center A is the tangent plane of the surface V_r (V_r is basic surface of equipped zone $V_{r(m)}(H)$), plane $(M(A))$ is the tangent m-plane of the basic surface in the center A. The hyperplane $H(A)$ is the equipping plane of the zone $V_{r(m)}(H)$. At that time we suppose, that the condition of the incidence of planes Λ, M, H is executed .

On the other hand, equations (2), (4) determine in the frame R^0 the hyperplane H_r [9], and equations (3) characterize an equipment of the hyperzone H_r by field of planes M. This field of planes M is determined by the field of the geometrical object $\{M^{\hat{\alpha}}_a\}$ – field of the fundamental equipping object of the hyperzone H_r. We will disignate the hyperzone H_r, which is equipped by the field of planes M, as $H_r(M)$. Thus, the theory of the three-component $\mathcal{H}$-distribution is a generalization of theories of the regular hyperzone H_r and the zone $V_{r(m)}(H)$ of the affine space.

4 Tensor of inholonomicity of equipping distributions

Let consider the system of differential equations

$$\omega_0^{\hat{\alpha}} = M_a^{\hat{\alpha}} \omega^a, \tag{5}$$

which is associated with the M-distribution. This system is fully integrable if and only if, when the tensor of inholonomicity $\{r_{ab}^{\hat{\alpha}}\}$ of the equipping M-distribution

$$r_{ab}^{\hat{\alpha}} = \frac{1}{2}(M_{ab}^{\hat{\alpha}} - M_{ba}^{\hat{\alpha}})$$

equals to zero.

At $r_{ab}^{\hat{\alpha}} = 0$ the system (5) determines $(n-m+1)$-parametric assemblage of the m-dimensional surfaces V_m – m-dimensional integral manifolds. One and only one such manifold passes across each point of the area of such manifolds (planes M are rounded by m-dimensional surfaces V_m of $(n-m+1)$-parametric assemblage).

In the time of displacement of the center A along the fixed surface V_m equations, that determine the $\mathcal{H}$-distribution, define the tangent r-equipped surface $V_{m(r)}$ [4], which is equipped by the field of tangent hyperplanes H. Actually, from system, which consists from differential equations (5) and equations, which determines the $\mathcal{H}$-distribution, we can pick out a subsystem

$$\omega^{\hat{\alpha}} = M_b^{\hat{\alpha}} \omega^b, \quad \triangle M_a^{\hat{\alpha}} = M_{ab}^{\hat{\alpha}} \omega^b, \qquad \triangle \Lambda_p^i = \Lambda_{pb}^i \omega^b, \quad M_{[ab]}^{\hat{\alpha}} = 0.$$

This subsystem determines the tangent r-equipped surface $V_{m,r}$ [4]. In this case the geometrical object $\{H_\tau^{n+1}\}$ (object H) is the fundamental equipping object of the tangent r-equipped surface $V_{m,r}$. Such tangent r-equipped surfaces $V_{m,r}$, which are equipped by the field of tangent hyperplanes , we will disignate as $V_{m,r}(H)$. Thus, if the tensor of the inholonomicity $\{r_{ab}^{\hat{\alpha}}\}$ of the equipping M-distribution equals to zero, so the space A_{n+1} disintegrates to $(n-m+1)$-parametric assemblage of manifolds look as follows $V_{m,r}(H)$.

On the other hand, the $\mathcal{H}$-distribution for which $r_{ab}^{\hat{\alpha}} = 0$ can be interpreted like the hyperzone H_m, which is equipped by the field of tangent planes Λ. Hence, geometry of the $\mathcal{H}$-distribution of the affine space, naturally, is richer than geometry of tangent r-equipped surfaces and geometry of hyperzones H_m of the affine space, because it consists of a constructions, which don't have any sense for the latter. Also, geometry of the $\mathcal{H}$-distribution can be used for studying of degenerated hyperzones [6] and surfaces [1].

The system of differential equations

$$\omega^{n+1} = H_\tau^{n+1} \omega^\tau, \tag{6}$$

which is associated with the equipping distribution of hyperplanes H (H-distribution), is fully integrable if and only if, when the tensor of the first order

$$r_{\tau\sigma}^{n+1} = \frac{1}{2}(H_{\tau\sigma}^{n+1} - H_{\sigma\tau}^{n+1})$$

turns into zero.

On the condition, that the tensor of the inholonomicity $\{r^{n+1}_{\tau\sigma}\}$ of the equipping H-distribution equals to zero, the system (6) determines one-parametric assemblage of hypersurfaces V_n (planes H are rounded by hypersurfaces V_n of one-parametric assemblage).

In the time of a displacement of the center A along the fixed surface V_n equations, which determine the $\mathcal{H}$-distribution, represent equations of the hypersurface, which is equipped by fields of geometrical objects $\{\Lambda^{\hat{u}}_p\}$ and $\{M^{\hat{\alpha}}_a\}$ (fields of planes Λ and M, where $\Lambda \subset M$). Hence, the theory of the three-component $\mathcal{H}$-distribution is also the generalization of the theory of hypersurfaces of the affine space.

5 Normals of the equipping distributions

Quasi-tensors were constructed in the second differential neighborhood:

$$B^p = -\frac{1}{r+2}a^{pq}B_q, \quad B^i = -\frac{1}{m-r+2}a^{ji}B_j - \frac{1}{m-r+2}\Lambda_{pk}a^{ki}B^p,$$

$$B^\alpha = -\frac{1}{m-r+2}(H^{\gamma\alpha}B_\gamma + \Lambda_{p\gamma}H^{\gamma\alpha}B^p + M_{i\gamma}H^{\gamma\alpha}B^i),$$

$$\nabla B^p - B^p\omega^{n+1}_{n+1} + \omega^p_{n+1} = B^p_{\mathcal{K}}\omega^{\mathcal{K}},$$

$$\nabla B^i - B^i\omega^{n+1}_{n+1} + \omega^i_{n+1} = B^i_{\mathcal{K}}\omega^{\mathcal{K}}, \qquad \nabla B^\alpha - B^\alpha\omega^{n+1}_{n+1} + \omega^\alpha_{n+1} = B^\alpha_{\mathcal{K}}\omega^{\mathcal{K}}.$$

The geometrical object $\{B^\sigma\}$ determines the normal of the first kind of the $\mathcal{H}$-distribution by an inner invariant method. The normal B coincides with the Blaschke normal in case of the hyperplane distribution. Affine normals of the first kind B_{n-r+1}, B_{n-m+1} of the Λ-distribution and of the M-distribution accordingly are determined in the same way.

Quasi-tensors were constructed in the differential neighborhood of the second order:

$$\gamma^p = -\frac{1}{r+2}\Lambda^{pq}\gamma_q, \quad \gamma^i = -\frac{1}{m-r+2}M^{ji}\gamma_j + \frac{m-r-2}{m-r+2}\Lambda_{pk}M^{ki}\gamma^p,$$

$$\gamma^\alpha = -\frac{1}{n-m+2}(H^{\alpha\beta}\gamma_\beta + \frac{n-m-2}{n-m+2}(\Lambda_{p\gamma}H^{\alpha\gamma}\gamma^p + M_{i\gamma}H^{\alpha\gamma}\gamma^i),$$

$$\nabla\gamma^p - \gamma^p\omega^{n+1}_{n+1} + \omega^p_{n+1} = \gamma^p_{\mathcal{K}}\omega^{\mathcal{K}},$$

$$\nabla\gamma^i - \gamma^i\omega^{n+1}_{n+1} + \omega^i_{n+1} = \gamma^i_{\mathcal{K}}\omega^{\mathcal{K}}, \quad \nabla\gamma^\alpha - \gamma^\alpha\omega^{n+1}_{n+1} + \omega^\alpha_{n+1} = \gamma^\alpha_{\mathcal{K}}\omega^{\mathcal{K}}.$$

Fields of the geometrical objects $\{\gamma^a\}, \{\gamma^\sigma\}$ determine fields of the normals of the first kind of the equipping M-distribution, of the equipping H-distribution accordingly.

The quasi-tensor $\{\mathcal{M}^\sigma\}$:

$$\mathcal{M}^\sigma = \frac{1}{2}(\mathcal{L}^\sigma + \gamma^\sigma), \quad \nabla\mathcal{M}^\sigma - \mathcal{M}^\sigma\omega^{n+1}_{n+1} + \omega^\sigma_{n+1} = \mathcal{M}^\sigma_{\mathcal{K}}\omega_{\mathcal{K}},$$

determines the normal of the first kind of the $\mathcal{H}$-distribution in the differential neighborhood of the second order, which is invariant relatively of the projective group of the transformations.

The normal $\{\mathcal{M}^\sigma\}$ is the Mihajlesku normal of the first kind of the hyperplane distribution of the affine space.

The field of the affine normal of the first kind of the H-distribution is determined by the object $\{\hat{B}^\tau\}$ in the differential neighborhood of the third order:

$$\hat{B}^\tau = H^{\rho\tau}\hat{B}_\rho, \quad \nabla\hat{B}^\tau - \hat{B}^\tau\omega_{n+1}^{n+1} + \omega_{n+1}^\tau = \hat{B}^\tau_\mathcal{K}\omega^\mathcal{K}.$$

The quasi-tensor $\{\mathcal{Z}^\sigma\}$ of the third order:

$$\mathcal{Z}^\sigma = \hat{B}^\sigma + \hat{h}^\sigma, \quad \nabla\mathcal{Z}^\sigma - \mathcal{Z}^\sigma\omega_{n+1}^{n+1} + \omega_{n+1}^\sigma = \mathcal{Z}^\sigma_\mathcal{K}\omega^\mathcal{K},$$

determines the projective normal—analog of the Fubiny's normal for the H-distribution in each center of the forming element of the $\mathcal{H}$-distribution.

The object $\{\mathcal{Z}^a\}$ determines the projective normal of the first kind of the M-distribution.

The object $\{S^a\}$, where

$$S^\sigma = -\frac{1}{2}(H_{\rho n+1} + \frac{1}{n+2}p_\rho)H^{\rho\sigma}, \quad \nabla S^\sigma - S^\sigma\omega_{n+1}^{n+1} + \omega_{n+1}^\sigma = S^\sigma_\mathcal{K}\omega_\mathcal{K},$$

determines the projective normal of the first kind of the M-distribution.

Theorem 1 *The projective normals of the first kind $\mathcal{M}$, $\mathcal{Z}$, S determine bunches of the projective normals of the first kind of the $\mathcal{H}$-distribution:*
a) in the differential neighborhood of the second order

$$\tilde{M}^\sigma(\mathcal{E}) = \mathcal{M}^\sigma - \mathcal{E}(\mathcal{M}^\sigma - S^\sigma);$$

b) in the differential neighborhood of the third order

$$\hat{\Phi}^\sigma(\mathcal{E}) = \mathcal{Z}^\sigma - \mathcal{E}(\mathcal{Z}^\sigma - \mathcal{M}^\sigma), \quad \hat{\mathcal{Z}}^\sigma(\mathcal{E}) = \mathcal{Z}^\sigma - \mathcal{E}(\mathcal{Z}^\sigma - S^\sigma),$$

where $\mathcal{E}$ – absolute invariant.

These normals determine bunches of the projective normals of the first kind of the equipping M-distribution:
a) in the differential neighborhood of the second order

$$\tilde{M}^a(\mathcal{E}) = \mathcal{M}^i - \mathcal{E}(\mathcal{M}^a - S^a).$$

b) in the differential neighborhood of the third order $\{\hat{\Phi}^a(\mathcal{E})\}$, $\{\hat{\mathcal{M}}^a(\mathcal{E})\}$.

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