

# The Converse of Kelly's Lemma and Control-classes in Graph Reconstruction

*To Professor Adriano Barlotti on the occasion of his 80th birthday*

PAOLO DULIO<sup>1</sup>, VIRGILIO PANNONE<sup>2</sup>

<sup>1</sup>*Dipartimento di Matematica "F. Brioschi", Politecnico di Milano, Piazza Leonardo da Vinci 32, I-20133 Milano  
e-mail: paodul@mate.polimi.it*

<sup>2</sup>*Dipartimento di Matematica "U. Dini", Univeristà di Firenze, Viale Morgagni 67/A, I-50134 Firenze  
e-mail: pannone@math.unifi.it*

(Received July 26, 2004)

## Abstract

We prove a converse of the well-known Kelly's Lemma. This motivates the introduction of the general notions of  $\mathcal{K}$ -table,  $\mathcal{K}$ -congruence and control-class.

**Key words:** Graph; Kelly's Lemma; Reconstruction.

**2000 Mathematics Subject Classification:** 05C60

## 1 Introduction

An *Ulam-subgraph* of a (finite, simple, undirected, labelled) graph  $G$  of order  $n$  is a subgraph of order  $n - 1$  obtained from  $G$  by deleting a vertex of  $G$  and the edges incident to it. Such a subgraph can also be defined as a maximal induced subgraph of  $G$  or, simply, as a subgraph induced by  $n - 1$  vertices of  $G$ .

Thus, a graph  $G$  of order  $n$  gives rise to  $n$  distinct Ulam-subgraphs, the set of which is sometimes called the Ulam-deck of  $G$ . We shall denote by  $G^{(v)}$  the Ulam-subgraph of  $G$  obtained by deleting the vertex  $v$  of  $G$ . Note that distinct Ulam-subgraphs may be isomorphic.