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Directoids with Sectionally Switching Involutions*

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Abstract

It is shown that every directoid equipped with sectionally switching mappings can be represented as a certain implication algebra. Moreover, if the directoid is also commutative, the corresponding implication algebra is defined by four simple identities.

Key words: Directoid; commutative directoid; semilattice; involution; implication algebra; sectionally switching mapping.

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The concept of directoid was introduced by J. Ježek and R. Quackenbush [4] in the sake to axiomatize algebraic structures defined on upward directed ordered sets. In certain sense, directoids generalize semilattices. For the reader convenience, we repeat definitions and basic properties of these concepts.

An ordered set $(A;\leq)$ is *upward directed* if $U(x,y)\neq\emptyset$ for every $x,y\in A$, where $U(x,y)=\{a\in A;\ x\leq a \text{ and } y\leq a\}$. Elements of $U(x,y)$ are referred to be common upper bounds of x,y. Of course, if $(A;\leq)$ has a greatest element then it is upward directed.

Let $(A;\leq)$ be an upward directed set and $\sqcup$ denots a binary operation on A. The pair $\mathcal{A}=(A;\sqcup)$ is called a *directoid* if

(i) $x\sqcup y\in U(x,y)$ for all $x,y\in A$;

(ii) if $x\leq y$ then $x\sqcup y=y$ and $y\sqcup x=y$.

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