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Deductive Systems of BCK-Algebras

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Abstract

In this paper we shall give some results on irreducible deductive systems in BCK-algebras and we shall prove that the set of all deductive systems of a BCK-algebra is a Heyting algebra. As a consequence of this result we shall show that the annihilator F^* of a deductive system F is the the pseudocomplement of F. These results are more general than that the similar results given by M. Kondo in [7].

Key words: BCK-algebras, deductive system, irreducible deductive system, Heyting algebras, annihilators.

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1 Introduction and preliminaries

In [7] it was shown that the set of all ideals (or deductive systems, in our terminology) of a BCK-algebra $\mathbf{A}$ is a pseudocomplement distributive lattice and that the annihilator F^* of a deductive system F of $\mathbf{A}$ is the pseudocomplement of F. Related results on annihilators in Hilbert algebras and Tarski algebras (or also called commutative Hilbert algebras [6] or Abbot's implication algebras) are given in [2] and [3]. On the other hand, it was shown in [9] that the set of deductive systems $Ds(\mathbf{A})$ of a BCK-algebra $\mathbf{A}$ is an infinitely distributive lattice, and thus it is a Heyting algebra. In this note we will give a description of this fact and we shall prove that the annihilator F^* of the deductive system F can be obtained as $F^* = F \Rightarrow \{1\}$, where $\Rightarrow$ is the Heyting implication defined in the lattice $Ds(\mathbf{A})$.