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$2-(n^2, 2n, 2n-1)$ Designs Obtained from Affine Planes*

Andrea CAGGEGI

Dipartimento di Metodi e Modelli Matematici,
Facoltà di Ingegneria Università di Palermo,
viale delle Scienze I-90128, Palermo, Italy
e-mail: caggegi@unipa.it

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Abstract

The simple incidence structure $\mathcal{D}(\mathcal{A}, 2)$ formed by points and unordered pairs of distinct parallel lines of a finite affine plane $\mathcal{A} = (\mathcal{P}, \mathcal{L})$ of order $n > 2$ is a $2-(n^2, 2n, 2n-1)$ design. If $n = 3$, $\mathcal{D}(\mathcal{A}, 2)$ is the complementary design of $\mathcal{A}$. If $n = 4$, $\mathcal{D}(\mathcal{A}, 2)$ is isomorphic to the geometric design $AG_3(4, 2)$ (see [2; Theorem 1.2]). In this paper we give necessary and sufficient conditions for a $2-(n^2, 2n, 2n-1)$ design to be of the form $\mathcal{D}(\mathcal{A}, 2)$ for some finite affine plane $\mathcal{A}$ of order $n > 4$. As a consequence we obtain a characterization of small designs $\mathcal{D}(\mathcal{A}, 2)$.

Key words: $2-(n^2, 2n, 2n-1)$ designs; incidence structure; affine planes.

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By a $2-(v, k, \lambda)$ *design* we mean a pair $\mathcal{D} = (\mathcal{P}, \mathcal{B})$ where $\mathcal{P}$ is a set of v points and $\mathcal{B}$ is a collection of distinguished subsets of $\mathcal{P}$ called blocks such that each block contains k points and any two distinct points are contained in exactly λ common blocks[1]. Our main result is the following

Theorem 1 *Let n be an integer with $n > 4$ and let $\mathcal{D} = (\mathcal{P}, \mathcal{B})$ be a $2-(n^2, 2n, 2n-1)$ design. Then $\mathcal{D}$ is of the form $\mathcal{D}(\mathcal{A}, 2)$ if and only if the following two conditions are satisfied: (c_1) any three distinct points of $\mathcal{D}$*

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[1]For further definitions (and basic results) about 2-designs see [1].