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A Groupoid Characterization of Orthomodular Lattices *

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Abstract

We prove that an orthomodular lattice can be considered as a groupoid with a distinguished element satisfying simple identities.

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A bounded lattice is called an *ortholattice* if there is a unary operation $x \mapsto x^\perp$ called *orthocomplementation* such that
$x \vee x^\perp = 1$ and $x \wedge x^\perp = 0$ (i.e. $x^\perp$ is a *complement* of x)
$x^{\perp\perp} = x$ (it is an *involution*)
$x \leq y$ implies $y^\perp \leq x^\perp$ (it is *antitone*).
An ortholattice is thus considered as an algebra $\mathcal{L} = (L; \vee, \wedge, {}^\perp, 0, 1)$ of type $(2, 2, 1, 0, 0)$. Due to the above mentioned properties of orthocomplementation, it satisfies the De Morgan laws, i.e.
$(x \vee y)^\perp = x^\perp \wedge y^\perp$ and $(x \wedge y)^\perp = x^\perp \vee y^\perp$.
Hence, it can be considered also in the signature $(\vee, {}^\perp, 0)$ of type $(2, 1, 0)$ because $\wedge$ can be expressed by De Morgan laws as a term function in $\vee$ and ${}^\perp$ and $1 = 0^\perp$.

An ortholattice $\mathcal{L} = (L; \vee, \wedge, {}^\perp, 0, 1)$ is called *orthomodular* if it satisfies the implication
$x \leq y \Rightarrow x \vee (x^\perp \wedge y) = y$ (the *orthomodular law*)
which is equivalent to $x \leq y \Rightarrow y \wedge (y^\perp \vee x) = x$.

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